DEVELOPMENT AND APPLICATION OF A RATIONAL

WATER QUALITY PLANNING MODEL

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

The objective of this research was to develop and to demonstrate methodology for water quality planning and management for the case in which there is or could be interaction of pollutants in the stream.

In water pollution control, there are important objectives other than strict economic efficiency. However, it is felt that an orderly, rational, and economically efficient approach would be helpful in guiding the decisions of policy-making agencies.

The specific pollutants selected for this investigation were heat and BOD. The related standards were DO concentration, maximum allowable stream temperature, and allowable rise in stream temperature.

The determination of a minimum-cost abatement policy* for a particular set of conditions is of much value, but more information for policy guidance can be obtained by determining the policy response to changes in important factors, such as the deoxygenation and reaeration coefficients, streamflow, and quality standards.

Alternative sets of standards can be formulated, varying from rather stringent to very loose standards. Optimal policies for the basin can be determined for each of these sets. With such knowledge, a water quality agency could select the set of standards that provided the best combination of stream uses.

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*Throughout this report, "policy," when used in this sense, shall mean a schedule consisting of abatement levels for each waste type at each waste outfall.

A generalized stream was structured as an N-stage serial system, with principal waste outfalls, major tributaries, changes in standards, etc., constituting stage boundaries. Initial-value, two-dimensional dynamic programming was used to optimize the system and to determine the minimum-cost abatement policy which allowed specific standards to be attained. The two decision variables at each stage were (a) level of cooling of heated waste and (b) level of treatment of organic waste. The three state variables were (a) water temperature, (b) DO, and (c) BOD.

The Chattahoochee River basin was used as a numerical illustration of the approach and methodology developed in this investigation. Atlanta's Clayton wastewater treatment plant handles about 90 per cent of the organic waste that enters the river from Atlanta to a point about 100 miles downstream. In this section of the river, there are three steam-electric generating plants which use the river for condenser cooling water.

The river was modeled as a four-stage serial system. Optimal policies with regard to the abatement of thermal and organic wastes were determined for a wide range of (a) streamflow, (b) deoxygenation coefficient, (c) reaeration coefficient, and (d) water quality standards (DO, maximum temperature, and allowable temperature rise).

There are numerous limitations to the use of this methodology. Foremost among these are the quality and consistency of input data such as abatement costs, quality standards, streamflow, and waste production. The effect of such factors is presented.

The approach and methodology developed in this research make the economic effects of interdependent pollutants, water quality standards, and non-economic objectives more explicit to the end that water quality management programs may more effectively and efficiently serve the needs of society.

CHAPTER I

INTRODUCTION

Purpose and Objectives

The purpose of this research is to develop and to demonstrate through a numerical illustration the advantages of an improved approach to regional water quality planning and management. The specific objectives are (1) to minimize the total cost of treating organic and thermal wastes in a basin while satisfying multiple stream standards, and (2) to investigate the sensitivity of cost to changes in system parameters and water quality standards. The technique used employs dynamic programming with three state variables: water temperature, biochemical oxygen demand, and dissolved oxygen, and two decision variables: per cent treatment and per cent cooling. Both temperature and dissolved oxygen are constrained by stream standards.

Introductory Comments

Several investigators have studied the water quality management problem in recent years and have applied a variety of mathematical optimization methods, such as linear and dynamic programming. Although their purpose has been essentially the same as in the present investigation, i.e., the minimization of total quality control costs, they have dealt with a single quality parameter, represented by one quality standard. The parameter most often used has been dissolved oxygen concentration, with the requirement that it equal or exceed a certain value throughout the basin. In some cases, the standard may vary from segment to segment within the basin.

Dissolved oxygen has been the most critical quality parameter for many years in most water quality investigations. It is likely that it will continue to be one of the most important indicators of water quality. As urban and industrial expansion brings about quality problems of increasing magnitude and complexity, persons with planning and management responsibilities in the pollution control area are realizing that there are often several important quality parameters that must be considered jointly, because the parameters of interest are not independent. It is impossible to evolve an optimal* treatment schedule for one particular waste type in the basin to satisfy one parameter or standard without knowledge of or control over wastes and other factors that affect or influence the parameter in question. The instance in which there is only one principal type of pollutant and only one major quality parameter may be viewed as a special case of the more general situation. In earlier years, when this was usually the case, pollution was considered as biodegradable organic in nature; and dissolved oxygen was therefore a satisfactory indicator of stream quality.

Two specific water quality parameters have been selected for study in this investigation; they are dissolved oxygen (DO) and

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*An optimal treatment schedule will be considered to be the minimum cost schedule which satisfies all explicit water quality constraints related to the wastes being considered. It is assumed that considerations other than strict economic efficiency (such as industrial development and aesthetics) are incorporated in the quality standards themselves.

temperature (T). Dissolved oxygen and temperature are affected principally by organic and thermal wastes, respectively. The effect of organic wastes on a stream's oxygen resources has been studied for many years; however, stream temperature as a quality characteristic has been considered for a much shorter period. The dynamic effect of heat on other quality parameters, notably dissolved oxygen, has not been widely investigated to date. Heat dissipation and oxygen dynamics have been studied as relatively independent pollution problems in the past. Heat dissipation is, in fact, independent of dissolved oxygen considerations; but, when one considers that various components* of dissolved oxygen models are very much temperature dependent, it is apparent that the converse is not true.

Therefore, in a basin that has significant oxygen-demanding organic wastes and one or more major waste heat sources, it is undesirable to study the basin's thermal aspects separately and then select a constant mean temperature for independent consideration of the stream's dissolved oxygen resources. While this study is specifically concerned with dissolved oxygen and temperature as parameters, the general approach or framework of analysis should be valid for other combinations of interdependent quality parameters.

It is emphasized that, though specific parameter models have been selected for use in the over-all model, the focus of this study is the approach or framework of analysis which allows for and considers

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*e.g., K_1 and K_2, which are the deoxygenation and reaeration rate coefficients (1/days), respectively.

parameter interactions and seeks a minimum-cost solution which meets specified multiple constraints.

While it is recognized that there are oftentimes objectives other than strict economic efficiency in the area of pollution control, as in other areas of investment of a social or public nature, economic efficiency does, nevertheless, have much to recommend it. When one considers the funds that are to be expended in the pollution control area and the needs and demands for funds in competing areas of public investment, there is good justification for the use of an orderly, rational approach which is economically efficient to guide investment in water quality projects.

Alternative sets of water quality standards could be formulated, and a minimum-cost abatement program could be developed for each. These sets of standards should correspond to all types of use of the stream, from principally industrial (relatively loose standards) to recreation and conservation (relatively stringent standards).

In this way, an indication of the cost of meeting non-economic constraints may be obtained; and the proper policy-making groups, which would consider the interests and the needs of the public at large as well as those of industry and municipalities, could make sound decisions as to which set of standards resulted in the maximum net benefits to the particular basin. Both tangible and intangible benefits and costs could be considered for the specific basin.

The approach described herein should be regarded as a technique to provide guidance to policy makers. It makes the economic effects of interdependent pollutants, water quality standards, and non-economic objectives more explicit to the end that water quality management programs will more effectively and efficiently serve the needs of society.

Present Management Policy and Water Quality Standards

The objective of present water quality management policies and the various agencies that develop and implement them is to maintain satisfactory water quality in streams for the mutual benefit of all users, public and private. In a reasonably well developed river basin or region, the many users of water will have different quality requirements with regard to water; and this will be reflected in the importance that they place on the various quality parameters. Therefore, satisfactory quality is a variable concept; and management policies tend to become an attempt to satisfy the majority of users.

The difficulty encountered in meeting the objective of providing maximum benefit to the aggregate of water users is twofold. Decisions must be made concerning, first, what level of quality should be maintained in the streams, and, second, what is the best way to attain this quality level.

The following remarks by Edward J. Cleary in the preface to Kneese's *The Economics of Regional Water Quality Management* give some insight into and perspective on the above questions:

> On the one hand, there are some who would tolerate an attitude of unconcern about pollution until a nuisance is created. At the other extreme are those who assert that users of water should return it to streams as clean as "technically" possible. In between these viewpoints regarding the appropriate condition of streams--which range from acceptance of foulness to aspirations for pristine purity--an accommodation must be sought.

> We are not confronted with a question of absolutes--of clean water versus dirty water or of fish versus factories--but one of efficient adjustment to water reuse. A reasonable basis for decision-making is to be found in weighing the benefits and costs of maintaining various levels of river quality (18).

The usual result of interaction among the social, political, and economic factors and interests is the establishment of water quality standards. It is noted that standards reflect the changing needs and values of society; and, at the present time, the public is demanding a higher quality environment. This is perhaps a result of such factors as an improved standard of living, more leisure time, and a greater appreciation for aesthetics. One of the results of this has been the establishment of more stringent water quality standards in the form of stream standards, effluent standards, or a combination of these.

Effluent standards frequently result in a requirement of equal treatment for all users. This is based on the notion that such a policy is fair and equitable to all concerned. Also, it is an expedient solution to a very complex problem.

It has been difficult to apply the equal treatment approach to wastes other than municipal and similar, relatively simple, oxygen-demanding wastes. If, for example, 85 per cent treatment of standard oxygen-demanding wastes is required in a basin, the equivalent to 85 per cent BOD removal for a complex industrial waste which is not characterized by BOD is difficult to determine. It is notable that thermal wastes have not been required to conform to the equal treatment standards where such standards are used, thus precluding possible economies for the basin as a whole.

Regardless of whether stream or effluent standards are used, the objective is to maintain acceptable water quality in the basin; and the problem of developing a waste treatment schedule for the basin must eventually be considered. This must be attempted by the individual users in the case of stream standards or by a regulatory agency in the case of effluent standards.

It is difficult to determine the effects of the various waste effluents on downstream water quality, as well as the resulting economic costs which accrue to others in the basin. This is especially true where there are numerous waste outfalls and more than one major kind of waste, as in the case of oxygen-demanding wastes and thermal wastes. Under such conditions, difficulties are encountered in developing an efficient treatment schedule for two major reasons. First, even for one waste type, in order to specify a level of treatment for an outfall, one must know what levels of treatment have been adopted for all upstream outfalls to know what the residual waste loading is at his particular outfall. When one considers the number of possible combinations that would have to be investigated to even approach a minimum-cost schedule, computational requirements obviously become prohibitive.

Secondly, some water quality parameters are interdependent, e.g., dissolved oxygen and temperature; just as the dischargers of one waste type are not and cannot be considered independent, neither can interacting wastes be considered separately, e.g., oxygen-demanding organic and thermal wastes. In addition to being confronted with the impossible necessity of knowing what treatment decisions have been adopted by other users that produce the same waste type, one must also have knowledge of treatment levels of other waste types in order to minimize cost.

Although the goal of present policies is to meet quality standards, this is not always being done in the most economically efficient manner. One would expect total waste treatment cost in a highly developed basin to be above that resulting from a minimum-cost approach. This is especially true when one considers the popularity of the equal treatment requirement which does not consider such important factors as physical location of the waste outfall and proximity of other users and waste discharges.

Offsite Costs or Externalities

One should consider the factors which necessitate optimization or minimization of treatment costs over the entire basin. If policies which let each waste discharger minimize his local treatment costs are adopted, a minimum-cost solution for the basin results if the waste outfalls are far enough apart for the stream to recover to equilibrium prior to reaching another user or waste discharger.* The discharger could determine the minimum required level of treatment for his waste to prevent a violation of standards in the stream reach between his outfall and the next, i.e., he will discharge the maximum permissible amount of waste to the stream. In this case the waste dischargers are

*Some uses do not normally have waste discharges associated with them; e.g., aesthetics.

essentially independent, and there is no residual or unstabilized waste entering the reach from upstream wastes nor is there any waste entering the subsequent reach.

When waste outfalls are close enough together for residual pollution to enter subsequent reaches, this independence disappears, and one has the problem considered earlier of not being able to make treatment decisions without knowledge of the decisions of others. There are now in effect technical or economic links among the waste dischargers. The waste dischargers would still like to minimize their individual costs by discharging as much waste as permissible to the stream. The specific location of the outfall along the stream now assumes great importance in that the user farthest upstream can still minimize his costs by adopting a policy of minimum treatment that will not violate the standards in the stream reach between his outfall and the next. For a user located a very short distance downstream from several major waste sources, it is likely that there will be little or no natural waste assimilative capacity available, and a very high treatment level will be required.

The economist's description of a process where one's activities result in costs being incurred by others is "technologic external diseconomies," and water pollution is a classic example. In order to accomplish an economically optimal waste treatment system in the social sense, each discharger must consider and be responsible for the total costs resulting from his waste, not only his local treatment costs, but also those induced elsewhere in the basin. Unless he does this, he will tend to minimize only his own costs, and the costs to various dischargers will be a function of their locational superiority in the basin.

It has been suggested by Kneese (18) that, for water quality management purposes, the entire basin be considered as a single firm which would serve to internalize the externalities, i.e., the management would now be responsible for effects of all wastes on all users. An over-all or global minimum cost could be sought for the basin instead of the numerous local minima. Economic trade-offs among treatment plants would be allowed, and economic efficiency could be attained.

It is argued by some that if they are required to consider offsite costs they will have to pass them on to the consumer in the form of higher prices. The consumer is already bearing the cost through higher prices paid for products and services provided by the adversely affected parties. The consumer would experience a net gain from economically efficient waste treatment policies which would seek to meet water quality standards at minimum cost.

The following remarks by Kneese (18) serve to emphasize the importance of externalities:

> *Failure of municipal and industrial waste dischargers to consider that subsequent water use may be made more expensive or foreclosed entirely by the discharge of their wastes is perhaps the basic element of the pollution problem.* . . . When the offsite costs are not considered, an excessive amount of waste tends to be deposited in receiving waters, little effort is made to treat waste water, to recover materials from waste water, or to design and operate industrial processes so as to conserve materials.

> *. . . A society that allows waste dischargers to neglect the offsite costs of waste disposal will not only devote too few resources to the treatment of waste but will also produce too much waste in view of the damage it causes.* In a general way this may be considered the rationale for some form of social or political intervention in waste disposal decisions.

Justification for Systems Approach

When one reflects upon such factors as externalities, it is apparent that rule-of-thumb approaches to water quality planning and management are not likely to yield minimum cost results for complex river basins. The task of developing a rational water quality model has been viewed by several researchers as a systems problem. The term "systems analysis" has been represented and interpreted in many different ways. Some previous work in the pollution control area has led many to believe that systems analysis is a panacea; consequently, it has been over-emphasized, too much is expected of the approach, and some individuals have now chosen to discount studies that use or claim to use the systems approach.

For the purpose of this research, the term "systems analysis" is considered to be an orderly process whereby a real-world system is removed from a larger system for study and analysis. The inputs to and outputs from the system being studied are defined, and the system is broken down into its components and sub-components until each can be adequately represented by a model.

The system that has been selected for study in this research is a component of larger systems; and the elements which have been designated components of the system studied are themselves even smaller systems. It is necessary to select a system level for a particular study; however, interaction within the larger system must be considered.

Unconstrained Versus Constrained Mathematical Optimization

Unconstrained mathematical optimization does not attempt to meet a specified set of water quality standards but seeks to maximize net benefits over costs in the basin with regard to water use and reuse. In order to do this, one must know the benefits that accrue to all users of water at various levels of quality; also, the costs to all users for treating or reducing waste discharges must be quantified. When an incremental increase in treatment at each site results in equal incremental increases in benefits and costs, the mathematically optimum abatement system has been achieved. While, in theory, this is what one should strive for, difficulties are encountered in quantifying the benefits and costs to all users.

The alternative is to express the public and private benefits in the form of quality standards and to minimize the cost of attaining the standards. Different sets of standards would correspond to different uses and levels of use. This is essentially the present approach. It is the responsibility of public officials or agencies to interpret the existing local forces and to temper them with judgment to the end that both long-run and short-run objectives of society are reflected in the standards. The importance of judgment factors is great throughout the entire process: determining the proper standards, construction of the necessary mathematical models, and, finally, the mathematical optimization itself.

Scope, Assumptions, and Limitations

The principal objective of this investigation is to develop an approach to water quality planning and management for the case where pollutants interact in the stream. The specific pollutants considered are thermal wastes (heat) and organic wastes (BOD). It is felt that the approach used is valid and would be applicable to any other pair of interacting wastes. The specific mathematical models which are used to describe the dissipation of heat and BOD, as well as other relationships, such as those which connect heat to BOD and dissolved oxygen, were selected from the literature. There are undoubtedly limitations associated with some of these models, and several of these limitations will be pointed out later.

To illustrate the approach which was developed, a specific river basin, that of the Chattahoochee River below Atlanta, Georgia, was selected. Because some of the models and relationships in the form used do not appear to be strictly applicable to the Chattahoochee River, no inference with regard to the basin should be drawn from the results of this application. The portions of this investigation which deal with the Chattahoochee River basin should be taken for what they are, i.e., an illustration of the use of the approach developed in this study.

If an investigation is to provide meaningful information which can serve as a basis for policy making, experience and sound engineering judgment must be used in the selection or validation of the relationships to be incorporated in the over-all basin model. The political and social environment and institutions of the particular river basin must also be considered if the results of the mathematical model building and optimization are to be useful.

The major assumptions used in the temperature and dissolved oxygen interrelationship formulation for a river basin are now presented. These assumptions will be discussed in more detail in subsequent sections of this report.

It is assumed that the classical Streeter-Phelps model (36) describes the dissolved oxygen response of a stream to organic wastes (BOD). In many complex systems, there are important factors which are not considered in the basic Streeter-Phelps model. There are extensions and modifications to the basic model which do consider several of these factors. Such modifications should be used where warranted.

It is assumed that the approach of Velz and Gannon (44) describes the dissipation of excess temperature in a stream. Important in this formulation is the concept of an equilibrium or natural water temperature which the stream approaches exponentially after being heated. For the Chattahoochee River basin, the Velz-Gannon model for equilibrium water temperature appears to be inadequate but is used for illustrative purposes.

The steady-state is assumed with regard to streamflow and the discharge of heat and BOD to the stream. The possible error of such an assumption is apparent and is probably appreciable in the case of the Chattahoochee River below Atlanta. This assumption is made for the sake of simplicity and ease of analysis and computation.

It is assumed that a stream can be represented as a series of reaches which are homogeneous with respect to hydrology and physical characteristics within each reach. No additional waste or flow increment is allowed to enter within a reach.

To summarize this section, it may be stated that some of the assumptions are rather severe; and several of the models and relationships used are not entirely adequate. The illustration which will be presented for the Chattahoochee River basin should be regarded as a numerical demonstration of the approach developed in this investigation. This approach is the contribution which is thought to hold much promise for the future.

CHAPTER II

LITERATURE REVIEW

For several decades, development of the nation's water resources has been, to varying degrees, "comprehensive" and "multi-purpose" in nature. However, until recently, the principal purposes were flood control, irrigation, navigation, hydroelectric power generation, and municipal and industrial water supply. None of these uses is directly affected by the over-all quality of the resource. Except for consideration of pathogenic organisms from domestic wastes and maintenance of enough dissolved oxygen to prevent anaerobic conditions, quality aspects were of secondary importance in comparison to quantity aspects. This was reflected in the literature until recent years. Thus, until quite recently, the objectives of water resources development were protection of human life, health, and property and the enhancement of national economic development and commerce.

Since World War II, rapid social, political, economic, technologic, and population changes have taken place in the United States which have led to an expansion of the purposes of development to include low-flow augmentation for quality control, recreation, protection of fish and wildlife, and preservation of areas of historic or scenic value. These recently emphasized purposes are all related to water quality and have made the design of water resources systems much more complex than before.

In 1955 the Harvard water resources research group began a consideration of "the full range of the political-economic-technologic process of investment decision-making in the water resources field." (26) This included water resources systems analysis. The accepted approach to water resources planning and management was influenced by the publication of the group's work in 1962 (26). Although it was oriented principally toward the traditional "quantity" purposes, it clearly demonstrated the utility of operations research techniques in the design of complex, multi-purpose, multi-constrained water resources systems. In 1963, the Harvard group issued a report which addressed itself specifically to the quality aspects (42).

Kneese's definitive consideration of water quality economics (18) and the accomplishments of the Harvard group have provided the stimulus for most recent research and development in the area of water quality management systems. A recent publication by Kneese and Bower (19) extends the earlier work of Kneese.

Though much significant work has been reported in the literature illustrating the application of techniques, such as linear, non-linear, and dynamic programming, queueing theory, and simulation to reservoir and waste treatment systems and hydrologic problems, the literature reviewed in the remainder of this chapter will, for the most part, relate directly to the subject of this investigation; that is, regional water quality planning and management.

Linear Programming Models

Among the first of the water quality systems analysts of the early 1960's were Thomann and Sobel, who applied "linear systems analysis" to the problem of estuarial water quality management (41). This was based on a mathematical model for dissolved oxygen presented by Thomann in a previous paper (38). Later, Thomann (39) was concerned with obtaining minimum-cost pollution control; and he presented an application to the Delaware Estuary which utilized a model for temporal and spatial variation of dissolved oxygen. He also studied the sensitivity of cost to the dissolved oxygen reaeration rate. In another work (40), Thomann points out the value of determining least-cost solutions for different levels of water quality. He warns, however, that though systems analysis techniques provide "meaningful results for the person concerned with designing a water pollution control plan for a specific river, they by no means eliminate the necessity for decision-making in the socio-political arena."

In 1965, Sobel (35) compared a minimum-cost linear programming formulation for regional water quality systems with the traditional uniform treatment approach. The traditional policy was formulated as a mixed-integer problem. He also formulated the maximization of the benefit-cost ratio as a linear programming problem.

Like Thomann, Deininger (7) separated a river basin into reaches bounded by waste dischargers and used linear programming to determine treatment levels at each outfall. This procedure minimized total basin costs associated with maintaining quality standards. The cost of this solution was compared with the cost resulting from uniform treatment requirements.* He concluded that the linear programming formulation was much more efficient economically.

Rogers and Gemmell (33), using Deininger's formulation, considered low-flow augmentation of the Fox River in Wisconsin and Illinois. Levels of waste treatment in the basin were determined to meet quality constraints and to minimize regional cost. For successive increments of flow augmentation, marginal costs were determined; from this analysis, the optimal degree of augmentation was found.

Goodman and Dobbins (14) considered the problem of regional cost associated with use of a river for municipal water supply, assimilation of treated waste, and recreation. They developed a steady-state interrelationship model describing a river basin and varied waste treatment input data. From the results of successive trials, improved plans could be developed according to the policy objectives regarding water quality, costs, and benefits. They then incorporated an optimization routine based on the method of "steepest ascent." Goodman and Dobbins also concluded that a desired level of water quality can be maintained at lower total cost by allowing different levels of waste treatment than by the uniform treatment policy.

Johnson (15) also used linear programming to specify waste treatment requirements among dischargers on the Delaware Estuary. Four water

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*Though it is possible for the minimum cost and uniform treatment approaches to yield the same waste treatment requirements in a basin, this would not be anticipated in a basin containing numerous interdependent waste sources that could induce diseconomies.

quality management approaches were studied: (a) a traditional uniform treatment policy, (b) least-cost policy based on marginal cost considerations, and two effluent charge policies, (c) a single charge throughout the estuary and (d) varying charges in different reaches of the estuary. Results indicated that effluent charges could result in improved quality at a cost approaching the least-cost plan, and that the charges would not likely result in any major regional economic readjustments. It also appeared that an effluent charge would be more desirable from equity considerations. The most significant advantage for the uniform treatment approach was its simplicity with regard to data and administration requirements.

Loucks (23), ReVelle, Loucks, and Lynn (30), Loucks, ReVelle, and Lynn (25), and Loucks and Lynn (24) used linear programming to select waste treatment levels that would satisfy dissolved oxygen constraints at minimum regional cost. Their model was based on the classical Streeter-Phelps oxygen-sag equation (36). They also considered the sensitivity of cost and quality changes to streamflow as well as physical and economic parameters.

Non-linear and Dynamic Programming Models

Kerri (17) used non-linear programming to determine the minimum-cost solution to maintaining a specified dissolved oxygen concentration in Oregon's Willamette River basin. The minimum-cost solution with only the dissolved oxygen constraint yielded an annual treatment cost to the basin of $2,999,000.* Minimum-cost solutions with additional constraints were also determined. The requirements of primary and secondary treatment by all municipalities gave costs of $3,954,000 and $4,648,000, respectively. Uniform treatment by all municipalities and industries resulted in a cost of $4,733,000. In a subsequent paper (16), Kerri considered various aspects of implementing and operating regional waste treatment associations formed to take advantage of the significant cost savings.

Liebman (21) and Liebman and Lynn (22) used discrete dynamic programming to minimize total basin waste treatment costs associated with meeting specified dissolved oxygen stream standards for the Willamette River basin. It was felt that the use of the more realistic non-linear equations would yield an improvement over the linear approximation of earlier investigators. The optimal solution resulted in an annual basin cost of $2,970,000 as compared with $3,359,323 for a uniform treatment policy. It was found that alternate treatment schedules with costs very near the optimal would have significantly different required treatment levels. This was attributed to the "flatness" of the cost response surface.

In a recent paper, ReVelle, Loucks, and Lynn (29) compared the annual cost results from a linear programming formulation of the Willamette River basin with those obtained by Liebman. The results were $2,957,400 and $2,971,900 for the linear and dynamic programming

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*This cost is given as it was in the literature. It is doubtful that the number of significant figures is justifiable. This comment applies to cost figures in the remainder of this chapter.

formulations, respectively. The authors proceeded to discuss the utility of the dual variables in determining the saving which would result from a decrease in the dissolved oxygen standards in certain strategic river reaches.

Meier and Beightler (27) presented a method for decomposing non-serial river basin systems into equivalent serial systems which can be analyzed by dynamic programming. They consider that, since many waste treatment plants and reservoirs are located on tributary streams, the present limitation to serial, multistage systems is rather restrictive.

Other Water Quality Studies

One of the earlier regional water quality systems analysis studies was concerned with flow augmentation in the Willamette River basin (46,47,48). The analysis started at the uppermost reach in the basin and proceeded downstream, checking for a violation of the dissolved oxygen standard. When a violation was encountered, the model determined what streamflow was required to provide adequate dilution and increased assimilative capacity and sought this additional flow from upstream reservoirs. In this manner, a flow augmentation schedule was developed which would maintain the required quality level. With this knowledge, future water use could be planned so as not to conflict with quality goals.

Bramhall and Mills (3) considered the question of determining an optimal balance between waste treatment and low-flow augmentation. In addition, they discussed the advantages of effluent fees in improving stream quality.

Other recent publications indicative of the increasing emphasis on water quality management are concerned with pumped-storage hydroelectric power projects by Reynolds (31,32) and Velz, et al. (45) and irrigation systems by Orlob and Woods (28).

CHAPTER III

DISSOLVED OXYGEN AND TEMPERATURE RELATIONSHIPS IN FLOWING STREAMS

Before approaching the task of regional water quality planning and management, one must develop models or mathematical representations of the in-stream processes that relate to the water quality parameters of interest. These models should express the spatial and/or temporal response of the quality parameters to waste inputs for a stream reach.

In this investigation, dissolved oxygen (DO) and water temperature (T) are the quality parameters used. The corresponding wastes considered are characterized as biodegradable organic and thermal. The development of temperature and dissolved oxygen component models for the subsequent optimization algorithm is presented below.

Temperature Relationships

As pointed out previously, temperature is independent of dissolved oxygen. For this reason, temperature will be considered first.

There are two principal approaches to determining water temperature at a point downstream from an initial point where conditions are known. These are, first, the "energy budget" approach and, second, exponential dissipation of temperature approaching an equilibrium water temperature.

The energy budget approach predicts downstream water temperature from a consideration of the net exchange of heat between the mass of water in a stream reach and its environment. Typical of the application of this approach is the work of Schroepfer, et al. (34), who used the following equation to describe the process

$$T_A + \Delta T_A + \Delta T_S - \Delta T_E - \Delta T_C - \Delta T_R = T_B \tag{1}$$

in which

T_A = temperature of river at upstream point A

ΔT_A = temperature increase due to thermal addition

ΔT_S = temperature increase due to solar radiation

ΔT_E = temperature decrease due to latent heat loss

ΔT_C = temperature decrease due to convective heat loss

ΔT_R = temperature decrease due to thermal radiation

T_B = temperature of river at downstream point B.

Upon substitution of suitable expressions for the terms in Equation 1, Schroepfer obtained the following:

$$T_A + \left(\frac{0.1855}{Q}\right)H_A + \left(\frac{0.00445\ A}{Q}\right)\Big(H_S - 0.3253(10 + W) \tag{2}$$

$$(V_W - V_A) - 0.16(5 + W)(T_W - T_A) - 1.1(T_W - T_A)\Big) = T_B$$

in which

Q = river discharge, cfs

H_A = thermal addition, mega Btu/day

A = water surface area, 1000 sq.ft.

H_S = solar radiation, Btu/sq.ft./hr.

W = wind velocity, mph

V_w = vapor pressure corresponding to the mean temperature of the water surface, mm of Hg

V_a = partial pressure of water vapor at the temperature and relative humidity of the surrounding air, mm of Hg

T_w = mean water temperature, degrees farenheit, °F

T_a = mean air temperature, °F

T_A and T_B as before.

The exponential dissipation approach is based on the concept of an equilibrium water temperature. The differential between actual water temperature at a point and the equilibrium temperature is the driving force* in a first-order differential equation of the form

$$\frac{d(T - E)}{dt} = - K(T - E) \tag{3}$$

which, when integrated,

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*The driving force is considered to be the excess of water temperature over the equilibrium temperature. Therefore it will be positive after waste heat is added to a stream but could be negative; for example, below a reservoir discharging cold water from the hypolimnion. In either case, water temperature will approach the equilibrium temperature exponentially.

$$\int_{T_1}^{T_2} \frac{d(T - E)}{(T - E)} = -K \int_{t_1}^{t_2} dt \qquad (4)$$

yields

$$(T_2 - E) = (T_1 - E)e^{-K(t_2 - t_1)} \qquad (5)$$

or, after rearrangement

$$T_2 = E + (T_1 - E)e^{-K(t_2 - t_1)} \qquad (6)$$

in which

T = water temperature, °F

E = equilibrium water temperature, °F

$(T - E)$ = temperature excess, °F

t = time, days

K = temperature dissipation rate coefficient, 1/days

T_1 = water temperature at upstream point, °F

T_2 = water temperature at downstream point, °F

t_1 = time at upstream point

t_2 = time at downstream point

$(t_2 - t_1)$ = time of flow between points

$(T_1 - E)$ = temperature excess at upstream point, °F

$(T_2 - E)$ = temperature excess at downstream point, °F

e = Naperian base

Before this method may be used, one must determine a value for the equilibrium temperature. LeBosquet (20) assumed that water temperature tended to approach air temperature and obtained the following:

$$F = F_a \, 10^{-0.0102 \frac{KWD}{Q}} \tag{7}$$

in which

F = excess temperature of water over air at a distance D miles downstream, °F

F_a = initial excess temperature, °F

K = heat loss rate, Btu/ft^2/°F excess/hr.

W = average stream width, ft

D = distance to downstream point, mi.

Q = streamflow, cfs.

It is noted that the form of Equation 7 is the same as that of Equation 5.

While the approach of LeBosquet was useful, it was recognized that an equilibrium water temperature was a function of more than air temperature. Velz and Gannon (44) formulated an approximation of the equilibrium temperature as a function of air temperature, wind velocity, atmospheric vapor pressure, and solar radiation. All of these parameters were based on a statistical analysis of mean monthly values, and an appropriate recurrence interval could be selected to correspond to other risk factors in the study. For example, if one were using the seven-day, ten-year minimum streamflow in an investigation, one might select the once in ten-year meteorological parameters for a certain month.

For use in this investigation, monthly meteorological data was analyzed statistically as outlined in Velz and Gannon (44) so that appropriate approximations of equilibrium water temperature could be calculated. A computer program was developed to determine the response at selected points throughout a reach to a 1°F elevation in water temperature at the top of the reach for a particular streamflow and for a specific set of meteorological conditions. One may scale these "unit ordinates" by using a multiplier equal to the actual temperature rise at a thermal-waste outfall and obtain the temperature profile down the reach.*

The major assumptions of the heat dissipation model used will now be presented. The steady-state is assumed with regard to heat loading, streamflow, and meteorological conditions. Limitations to this assumption are apparent. Certain thermal power plants are used to generate peaking power; these are usually the older plants with high operating cost and lower efficiencies. Where hydroelectric facilities are used for peaking power purposes, streamflow will vary diurnally and throughout the week according to power requirements. Meteorological conditions vary seasonally, from day to day, and diurnally. However, since critical water quality conditions with respect to dissolved

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*The linear effect is justified due to the nature of the first-order or exponential relationship used. See Equation 3.

oxygen and temperature usually occur during dry, hot summer periods, the steady-state assumption is reasonable.

It has been assumed that the physical characteristics of the stream channel and the flow regime are constant within a reach, and that, within such a reach, heat dissipation is adequately represented by the exponential or first-order model. Also, only major or significant waste heat sources were considered. It is assumed that necessary solar radiation considerations were included in the equilibrium temperature calculation.

In general, the above conditions state that random or periodic fluctuations in system inputs were not considered. Appropriate input values were selected from the results of statistical analysis of available data. It was felt that for a regional water quality planning and management model such as developed in this study, this approach was both necessary from a computational standpoint and sufficient for obtaining information for policy guidance. These remarks will also apply to the assumptions regarding the dissolved oxygen model which is covered in the next section.

Dissolved Oxygen Relationships

The model used to describe the stream's dissolved oxygen response to inputs of biodegradable organic waste is based upon the Streeter-Phelps formulation (36) in which the following relationship for the time rate of change of the dissolved oxygen deficit was proposed

$$\frac{dD}{dt} = K_1 L - K_2 D \tag{8}$$

in which

$D = C_s - C$ = dissolved oxygen deficit, milligrams/liter, (mg/ℓ)

t = time, days

K_1 = deoxygenation rate coefficient, 1/days

L = organic waste concentration, mg/ℓ

K_2 = reaeration rate coefficient, 1/days

which, when integrated, gives the classical Streeter-Phelps equation

$$D_t = \frac{K_1 L_a}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) + D_a e^{-K_2 t} \tag{9}$$

in which

D_t = DO deficit t days downstream from initial point, mg/ℓ

L_a = initial BOD concentration, mg/ℓ

D_a = initial DO deficit, mg/ℓ

t = time-of-flow between points, days

K_1, K_2 = deoxygenation and reaeration rate coefficients, respectively, 1/days

The assumptions and limitations of this model are numerous; it is of interest that they were presented and discussed by Streeter and Phelps in their 1925 report (36).

The basic assumption is that deoxygenation and reaeration are, in fact, first-order reactions. This is being challenged by numerous current investigators. It should be noted that, as one goes to more realistic, and, therefore, more complex, models, e.g., second-order kinetics, data requirements regarding various rate coefficients, waste characteristics, and biological populations increase greatly. This was the principal justification for selecting the basic Streeter-Phelps model. For regional planning purposes, one would not expect such information to be generally available. Where it is available, one might wish to use these more complex formulations.

It is further assumed that algal respiration and benthal demands either do not occur or may reasonably be neglected. Extensions to the basic Streeter-Phelps model which consider such phenomena exist and should be used when deemed necessary.

The steady-state is assumed with regard to organic waste inflows and streamflow. Channel and flow characteristics are considered constant throughout a stream reach.*

The deoxygenation rate coefficient, K_1, for a particular waste is considered to be a function only of temperature. The relatinship of K_1 to temperature (8) is described by

$$K_{1,T} = K_{1,20}\ (1.047)^{(T\ -\ 20)} \tag{10}$$

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*If one considers reaches within a river basin to be bounded only by waste outfalls, the possible error of this assumption is apparent. In a later section, confluence of a major tributary with the main stream and significant changes in flow regime will be discussed as delineating new reaches. Therefore, the assumption of constant flow and channel conditions within a reach should cause little difficulty.

in which

T = temperature, degrees Centigrade (°C)

$K_{1,T}$ = deoxygenation rate coefficient at T°C, 1/days

$K_{1,20}$ = deoxygenation rate coefficient at 20°C, 1/days

Since channel and flow conditions are considered to be constant within a reach, it is assumed that regardless of how it is determined initially, the reaeration coefficient, K_2, is a function only of temperature within each reach and that this relationship (5) is given by

$$K_{2,T} = K_{2,20} (1.024)^{(T - 20)} \tag{11}$$

in which

T = temperature, °C

$K_{2,T}$ = reaeration rate coefficient at T°C, 1/days

$K_{2,20}$ = reaeration rate coefficient at 20°C, 1/days.

The driving force in the Streeter-Phelps equation is the dissolved oxygen deficit, i.e., saturation dissolved oxygen minus actual dissolved oxygen. For this equation to be valid, the saturation dissolved oxygen must be constant along the stream reach. Tennessee Valley Authority personnel (37) have related saturation dissolved oxygen to temperature by using the following polynomial approximation:

$$C_s = a + b_1T + b_2T^2 + b_3T^3 \tag{12}$$

where:

T = temperature, °C

C_s = saturation concentration of dissolved oxygen at t°C, mg/ℓ

a, b_1, b_2, b_3 = constants

which, when evaluated, yielded

$$C_s = 14.652 - 0.41022T + 0.0079910T^2 - 0.000077774T^3 \qquad (13)$$

with a multiple correlation coefficient of 0.99980. Therefore, it is apparent that in a heated stream reach, the saturation dissolved oxygen concentration is not constant, but it approaches the saturation dissolved oxygen concentration corresponding to the equilibrium temperature as the water temperature approaches that equilibrium temperature.

Liebman (21) modified the Streeter-Phelps model to accommodate a linearly varying saturation dissolved oxygen concentration along a reach

$$C_s^t = C_s^o - Bt \qquad (14)$$

in which

t = time-of-flow to point of interest, days

C_s^t = saturation DO at time t downstream, mg/ℓ

C_s^o = initial saturation DO, mg/ℓ

$$B = \frac{C_s^o - C_s^T}{T} = \text{constant}$$

C_s^T = saturation DO at end of reach, mg/ℓ

T = total time-of-flow in the reach, days

The rate of change of the dissolved oxygen deficit with respect to time was then

$$\frac{dD}{dt} = K_1L_t - K_2D_t - B = K_1L_a\ e^{-K_1t} - K_2D_t - B \tag{15}$$

which, upon integration, gave

$$D_t = \frac{K_1L_A}{K_2 - K_1}\left(e^{-K_1t} - e^{-K_2t}\right) + \left(D_a + \frac{B}{K_2}\right) e^{-K_2t} - \frac{B}{K_2} \tag{16}$$

Though Liebman considered this variation of saturation dissolved oxygen due to changing temperature along the reach, he assumed that the rate coefficients, K_1 and K_2, were constant within a reach.

The actual variation in water temperature along the stream is continuous, hence the continuous variation in two of the parameters* of the dissolved oxygen model (Equation 9). Good approximation of the variation may be obtained by dividing the reach into sub-reaches of constant temperature; the shorter the sub-reaches, the closer the approximation. Using this approach, K_1, K_2, and the saturation dissolved oxygen are calculated on the basis of the average temperature

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*$K_1 = f(T)$, $K_2 = f'(T)$.

in the sub-reach; and the initial dissolved oxygen deficit, D_a, is computed as the saturation dissolved oxygen minus the initial dissolved oxygen. By using Equation 9, the dissolved oxygen deficit at the end of the sub-reach, D_t, is calculated. This D_t is then used as the initial dissolved oxygen deficit, D_a, in the next sub-reach, etc.

The rate of change of water temperature and the temperature-dependent parameters of the dissolved oxygen model decrease with time or distance down the reach. This indicates that the best balance between accuracy and amount of computation would be to use very short sub-reaches immediately downstream from a thermal-waste source, where temperature is decreasing most rapidly, and to increase the lengths of the sub-reaches as the water temperature curve flattens. The use of shorter sub-reaches below a waste source is also desirable because of the more rapid decrease in dissolved oxygen immediately below a waste source.

Additional small sub-reaches may be desired in the region of the oxygen "sag point" to provide better definition of its location. A useful guide to selection of these sub-reach lengths would be to have the same incremental decrease of water temperature in each; this would result in the effect of using the mean sub-reach temperature being the same in all sub-reaches. The decision as to the size of this temperature increment depends upon the precision desired and computer capabilities. In any case, allowing the parameters to respond dynamically to changing temperature will yield a more accurate representation of the real system.

In this chapter, the component models for the quality response of a single stream reach to thermal and organic waste loading have been presented and discussed. In summary, it is felt that the more realistic consideration of temperature-dependent parameters used in this investigation can result in an improved formulation for regional water quality planning and management where significant sources of thermal wastes exist.

CHAPTER IV

DESCRIPTION OF PHYSICAL MODEL

The objective of this investigation was to develop and to demonstrate a method for minimizing total river basin waste treatment costs associated with maintaining specified dissolved oxygen and temperature standards. In order to do this, the pertinent components of the real system were abstracted and used to form a physical model or system representation upon which a mathematical optimization model may be based.

The decision as to what components or factors of the real system are pertinent to the physical model and what, therefore, should be considered is a matter of judgment and depends on the system level being studied. In the broadest sense, many social, economic, and political factors have a bearing, direct or indirect, on a stream's water quality.* These are in addition to the obvious factors, such as waste discharges, streamflow, channel characteristics, and meteorological conditions, all of which can be quantified and all of which will be components of the physical model in this study.

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*It is beyond the scope of this investigation to consider these influences in detail. They are not disregarded, however, because they are considered to be represented in the water quality standards. Different combinations of these factors will result in alternate sets of water quality standards.

A generalized definition sketch of a river basin is shown in Figure 1 and serves to illustrate the transition from river basin to physical model required for the use of the optimization model.

Boundaries

First consideration must be given the matter of boundaries of the system. These are of three types: (a) the upstream limit of the system, (b) the downstream limit of the system, and (c) boundaries of individual reaches between (a) and (b).

Upstream Boundary Conditions

In the general case, the upstream boundary would be that location where the first significant* thermal or organic waste (BOD) source is encountered. Above this point, natural quality conditions exist, i.e., the dissolved oxygen would be expected to be high (e.g., 85-90 per cent of the dissolved oxygen saturation value), the BOD concentration would be low (e.g., BOD = 0.5-2 mg/ℓ) and would be due only to natural sources such as overland flow, and the water would be at the equilibrium temperature. If the entire river basin is in a single water planning jurisdiction or if basin planning is coordinated and can extend across jurisditions, this is the best method of determining the upper bound.

If this is not the case, then a jurisdictional boundary (e.g., a state line) may constitute the upper boundary; and initial quality

*In this and subsequent usage, "significant" indicates a condition that has or may have a distinct, measurable effect on basin water quality or economies. Obviously, what may be important in a localized situation may not be for the region or basin. The scope and goals of a particular study will dictate what is "significant" and what is not.

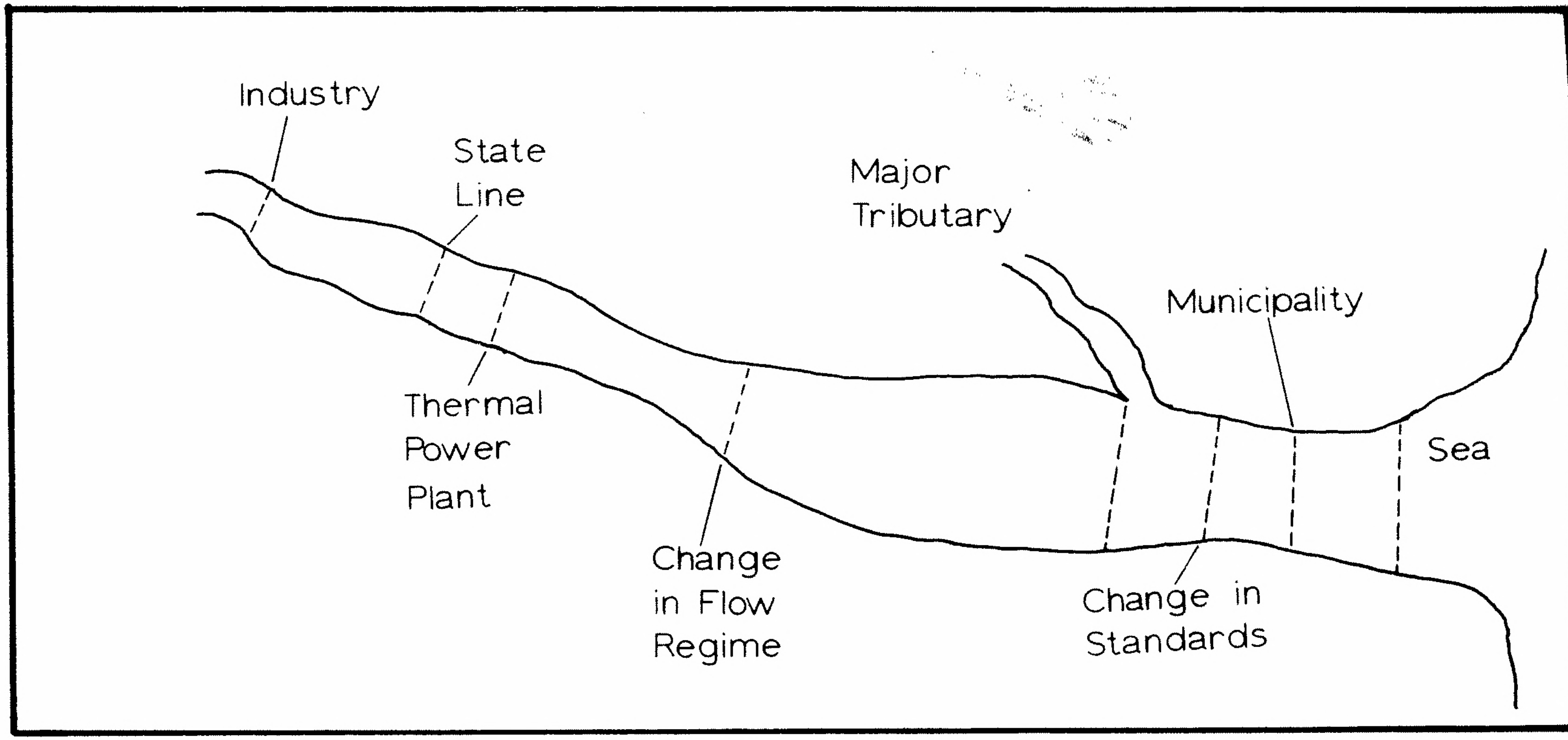

Figure 1. Definition Sketch of Generalized River Basin

conditions may be assumed to be those required by law (particularly in the case of interstate streams).

Downstream Boundary Conditions

There are several methods of locating the downstream boundary of the system. As above, a jurisdictional boundary may be indicated. If the river under study runs into a much larger river* or the sea, a discontinuity is created and could therefore be considered a proper lower bound.

A lower boundary would be indicated at a point where the stream's water quality becomes independent of upstream wastes; i.e., regardless of the abatement schedule adopted, quality would have completely recovered prior to reaching this point. This would require a relatively long reach below the last waste outfall.

Intermediate Reach Boundaries

Several types of intermediate reach boundaries are considered. Any significant source of organic waste constitutes the upper boundary of a new reach. This would include large municipalities as well as industries producing biodegradable organic wastes, such as paper, chemical, and food-processing facilities. Significant sources of thermal wastes, such as steam-electric generating plants and other industrial cooling water users would constitute reach boundaries.

Since it will be assumed that flow is constant within a reach,

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*In this situation, the first river would be considered to be a point source of flow and waste to a larger system, the basin of the larger river. This is the same approach that will be used for tributaries to the main stream in the basin of the river being studied.

major tributaries must start a new reach. If, however, the flow within a reach can be satisfactorily represented by its average flow, then it is not considered necessary to start a new reach at the confluence of the tributary and the main stream.

Significant changes in flow regime constitute reach boundaries. This is necessary because of the assumption of constant channel and flow characteristics within a reach. For example, if a stream flows from one physiographic region into another, one would expect the slope, depth, velocity of flow, width, etc., to change. In this case, the reaeration rate coefficient would be expected to change, since it is strongly affected by depth and velocity of flow. Man-made channel improvements could cause similar effects.

Demographic boundaries and changes in water quality standards should also be considered as reach boundaries. Most of these bounding conditions are illustrated in Figure 1.

Summarizing, appropriate upper and lower boundaries are applied to the river basin system. The system within these boundaries is divided into discrete reaches, a new reach starting wherever a significant waste or flow input is encountered or wherever constant conditions within reaches must be assured. Political factors, such as jurisdictional boundaries, and changes in water quality standards may also form reach boundaries.

Figure 2 shows the basin of Figure 1 as a series* of discrete

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*This investigation applies only to basins which can be adequately represented as serial reaches; i.e., no branching is allowed. If a polluted branch is encountered, one may wish to include it as a tributary, a point source of flow and waste.

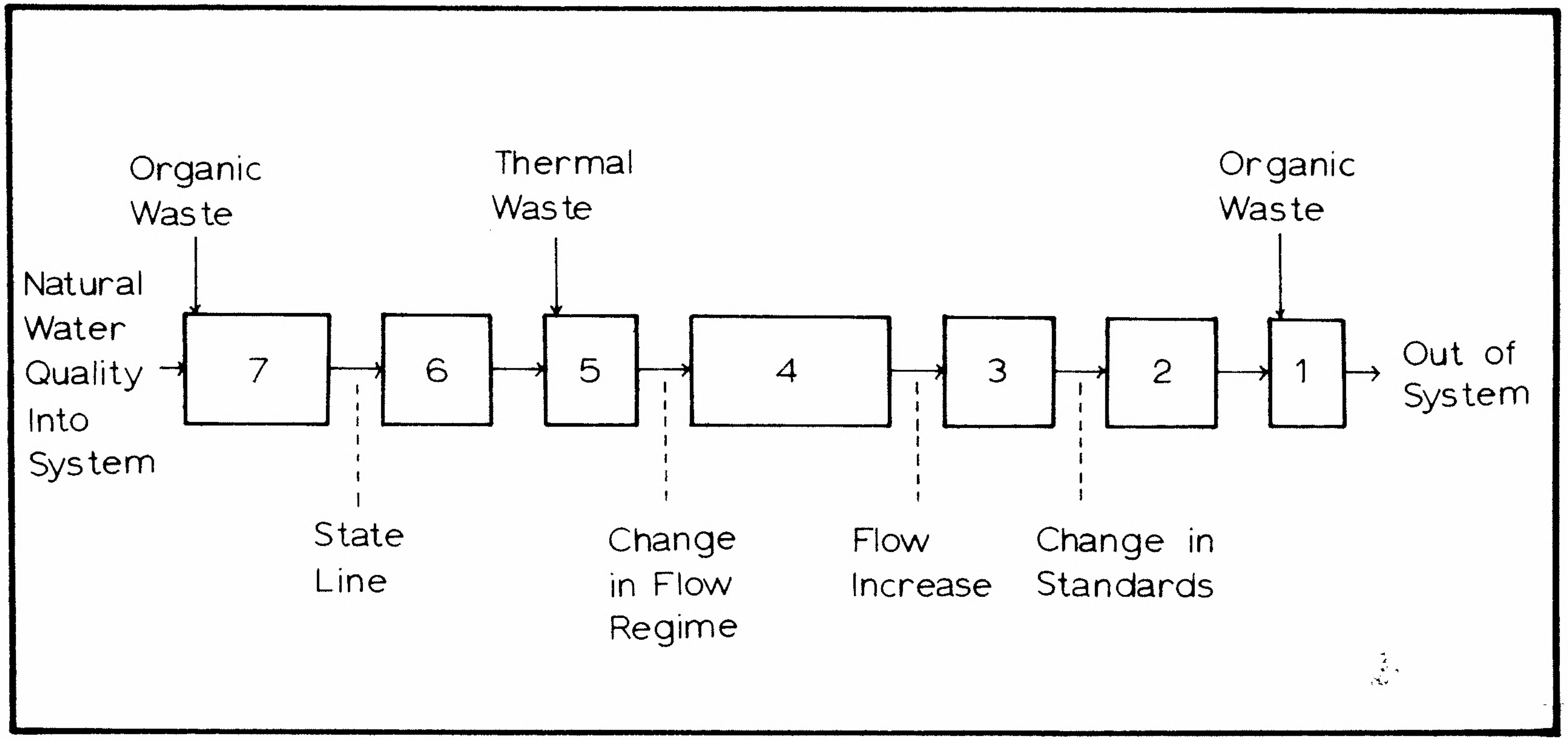

Figure 2. Generalized River Basin Shown as a Series of Discrete Steady-State Reaches

reaches* (or stages). It should be noted that the input to a reach is made up of the input to the system at that reach plus the output from the previous reach.

Streamflow

It has been assumed that the flow increases only at tributaries, that it is constant within a reach, and that it can be adequately described as the average of the flow into and out of a reach. Consideration of a flow hydrograph of the entire system for the design conditions will indicate the presence of any large step increases in flow due to inflows of large tributary streams and the necessity of creating new reaches to justify the above assumption.

It is also assumed that no change in streamflow occurs at waste outfalls. This precludes the possibility of inter-basin transfers of water. It is felt that this is a reasonable assumption, because most industrial process water is returned to the stream, and most municipal withdrawals return as sewage. In specific cases where this appears to be invalid, one may consider a waste outfall as a tributary with positive or negative flow.

Flow hydrographs for the system are generally developed from a statistical analysis of flow data for gaging sites operated by the U. S. Geological Survey or other governmental agency or private group.

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*The numbering system for the reaches is reversed from that normally used. This is due to the dynamic programming technique to be used. The reach farthest downstream is designated 1, the one farthest upstream, N. A generalized reach will be designated n.

From such an analysis, flow-duration relationships* may be obtained for discrete points corresponding to gage locations along the stream. Step increases in flow at tributaries may be estimated by apportioning the increase in flow between gaging stations to the tributaries according to their respective drainage areas.

It may be considered more meaningful to develop separate flow-duration relationships for each month of the year.** This would provide more correspondence with the statistical analysis of meteorological data.

Waste Inputs

Waste flows, both organic and thermal, are treated as deterministic. One may use either annual or seasonal mean inputs, whichever seems most appropriate to the system being studied. As with all system inputs, the degree of refinement is a function of the availability of data.

Instantaneous and complete mixing of both thermal and organic wastes is assumed for the sake of simplicity. Longitudinal dispersion is ignored. If, for example, thermal stratification were allowed, it would be necessary to split the flow into heated and unheated portions and to consider them separately. The increased complexity is apparent.

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*For example, the seven-day, ten-year low flow, which is the minimum flow that persists for seven days on the average of once in ten years.

**For example, the three-day, ten-year September low flow, which is the minimum flow that persists for three days in September on the average of once in ten years. In some instances, seasonal relationships might be appropriate.

Construction of a Generalized "N-Stage" Basin Model

Consideration of Figure 2 indicates the necessity of developing a generalized physical model for a basin. Some reaches have waste inputs and some do not; some reaches have treatment facilities and some do not.*

The generalized stream reach (or nth stage) used in this investigation is shown as Figure 3. Both organic and thermal wastes, designated as $X_{B,n}$ and $X_{T,n}$, respectively, may be discharged into the upper end of the reach, the actual amount depending upon the BOD and heat produced at the location, B_n and T_n, respectively, and the degree of treatment of each, $d_{B,n}$ and $d_{T,n}$, respectively.** If, for example, a steam-electric generating plant were discharging heat in its cooling water at the rate of 4×10^9 Btu/hr to the stream, T_n would equal 4×10^9 Btu/hr and $d_{T,n}$ could vary from 0 per cent to 100 per cent, so that $X_{T,n}$ could vary from four billion to zero Btu/hr. If this same plant discharged no organic waste (BOD), then a dummy waste would be used; i.e., B_n would be put into the reach at a value of zero. It would make no difference whether a range of 0 to 100 or 0 to 0 is used for $d_{B,n}$, because there would be no cost involved.

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*It is assumed that all municipal and industrial wastes are subject to abatement. It is possible, however, to have a tributary that is polluted from diffuse sources and therefore not subject to treatment.

**In all subscripted variables, n is the stage index. In the double-subscripted variables, X and d, the subscript B refers to BOD and T refers to thermal wastes. In the single-subscripted variables T and B, the T and B represent actual rates of production of thermal waste and BOD, respectively.

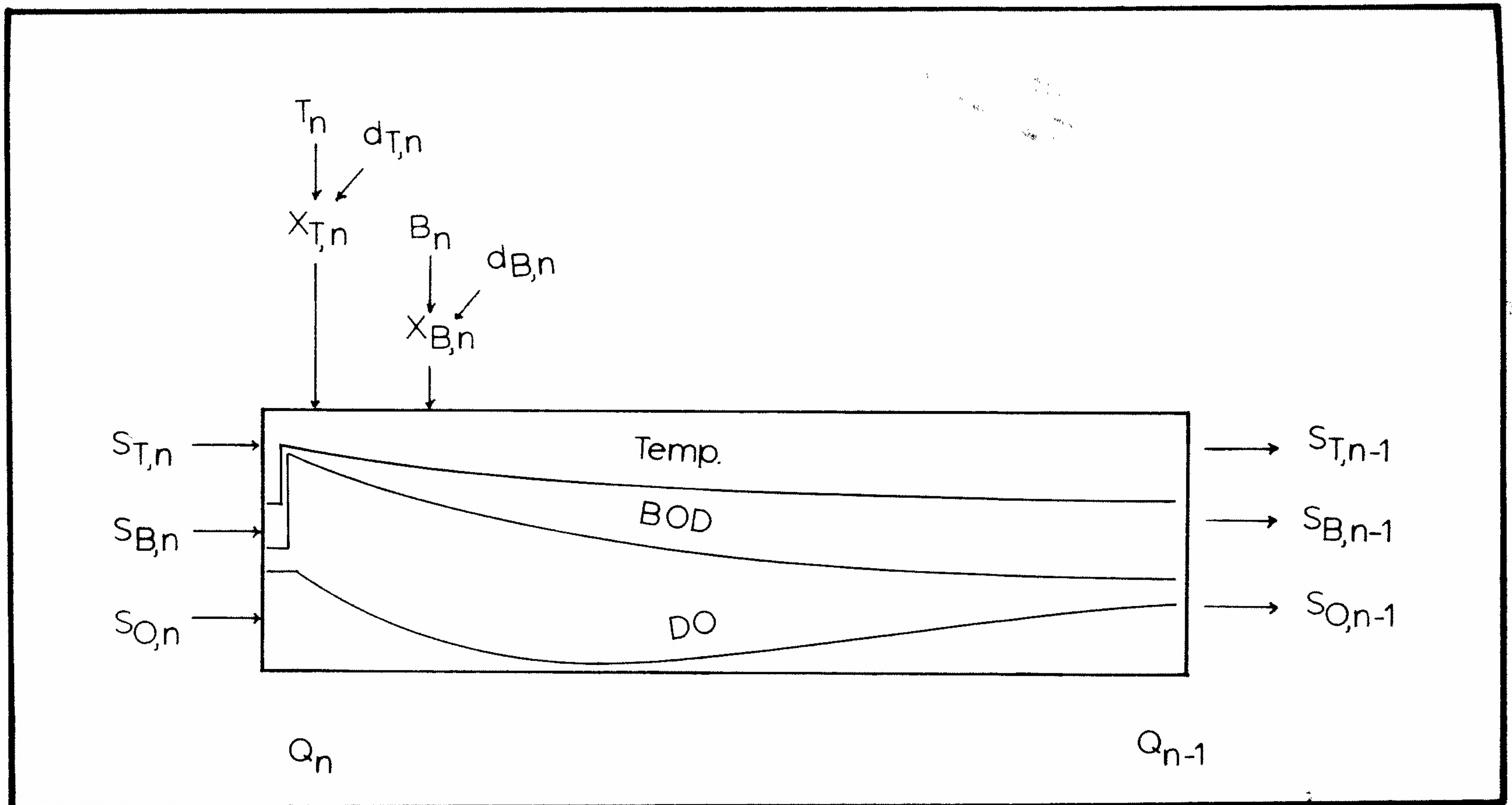

Figure 3. Generalized Stream Reach or Stage

The inputs to reach n from the previous reach are shown as $S_{T,n}$, $S_{B,n}$, and $S_{O,n}$, which are the incoming temperature (°F), BOD (mg/ℓ), and dissolved oxygen (mg/ℓ), respectively. These values, along with the waste added to the reach, determine the aggregate quality conditions at the top of the reach.* These relations are now presented.

The mean flow in the reach, $\bar{Q}_n$, is

$$\bar{Q}_n = \frac{(Q_n + Q_{n-1})}{2} \tag{17}$$

where Q_n and Q_{n-1} are the flows at the top and bottom of reach n.

The mixed temperature at the top of reach n, $TMIX_n$ (°F), is a function of the mean flow, $\bar{Q}_n$ (cfs), the incoming temperature from the previous reach, $S_{T,n}$ (°F), and the waste heat added, $X_{T,n}$ (Btu/hr)

$$TMIX_n = S_{T,n} + \frac{C_1(X_{T,n})}{\bar{Q}_n} \tag{18}$$

where C_1 is a unit conversion constant and is equal to 4.45×10^{-10} $(ft^3 hr)/(\text{pounds-seconds})$.

The mixed organic waste concentration at the top of reach n, ORG_n (mg/ℓ), is a function of the mean flow, the incoming waste concentration, $S_{B,n}$ (mg/ℓ), and the organic waste added, $X_{B,n}$ (pounds/day)

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*It is assumed that the dissolved oxygen concentration of heated or wastewater effluents does not significantly affect the mixed dissolved oxygen concentration at the top of a reach. If, in a specific instance, this assumption is not reasonable, one may elect to allow for other conditions.

$$ORG_n = S_{B,n} + \frac{C_2(X_{B,n})}{\bar{Q}_n} \tag{19}$$

where C_2 is a unit conversion constant equal to 1.86×10^{-7} $(ft^3 day)/$ (pounds-seconds).

After determining the dissolved oxygen saturation at the top of the reach, $DOSAT_n$, as a function of the mixed temperature, the dissolved oxygen concentration, DO_n (mg/ℓ), is related to the saturation dissolved oxygen concentration and the incoming dissolved oxygen concentration, $S_{O,n}$, as follows

$$DO_n = \begin{cases} DOSAT_n; & \text{if } S_{O,n} \geq DOSAT_n{}^{*} \\ S_{O,n}; & \text{if } S_{O,n} < DOSAT_n \end{cases} \tag{20}$$

This indicates that the dissolved oxygen deficit at the top of reach n, $DODEF_n$ (mg/ℓ), is

$$DODEF_n = \begin{cases} 0 & ; \text{if } S_{O,n} \geq DOSAT_n \\ DOSAT_n - S_{O,n}; & \text{if } S_{O,n} < DOSAT_n \end{cases} \tag{21}$$

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*This precludes the possibility of supersaturation.

Use of Parameter Transfer Functions

Since it has been assumed that wastes enter only at the top of the reach and that flow and channel conditions are constant throughout the reach, the transfer functions discussed in Chapter III may be used to determine the value of the quality parameters at the end of the reach. This is accomplished in a step-wise manner, by moving through one discrete sub-reach at a time until the entire reach has been traversed.

Because water temperature is independent of other quality parameters, the exponential heat dissipation model may be used to determine water temperatures at selected distances or flow-time increments, prior to considering dissolved oxygen. With this information, mean temperatures are calculated for the sub-reaches. Mean dissolved oxygen saturation values are calculated for each sub-reach; and the 20°C dissolved oxygen model rate coefficients, K_1 and K_2, are corrected for temperature.

The initial mixed parameter values (inflow from previous reach and waste inputs), $TMIX_n$, ORG_n, and DO_n, form the input vector to the first sub-reach. The Streeter-Phelps dissolved oxygen model is then used to determine the output vector from the sub-reach which, with new values of the rate coefficients and saturation dissolved oxygen, becomes the input vector to the second sub-reach. This process continues to the end of the reach. An example of the procedure is illustrated in Figure 4.

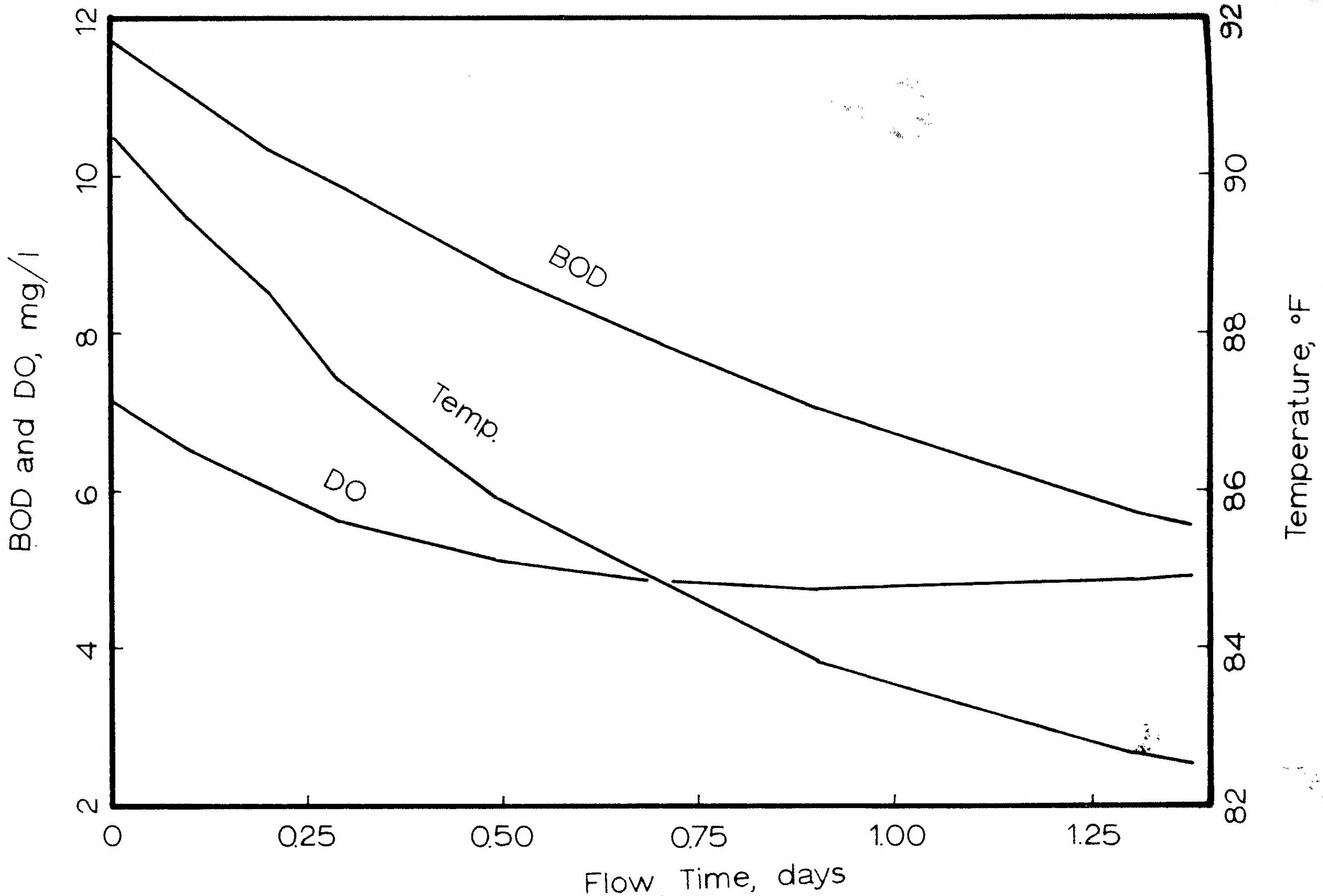

Figure 4. Typical Quality Parameter Profiles Resulting from use of Transfer Functions on Sub-Reaches

The individual, discrete series of reaches or stages may now be connected to form an N-stage river-basin, water-quality model for temperature and dissolved oxygen. This is shown in Figure 5. Given the physical conditions of the channel, meteorology, flow hydrograph, time-of-travel data, waste production, and abatement levels for conventional treatment of organic wastes and cooling of thermal wastes, profiles of temperature and dissolved oxygen, as well as other related parameters, such as BOD, saturation dissolved oxygen, dissolved oxygen deficit, and the rate coefficients, may be calculated along the entire basin.

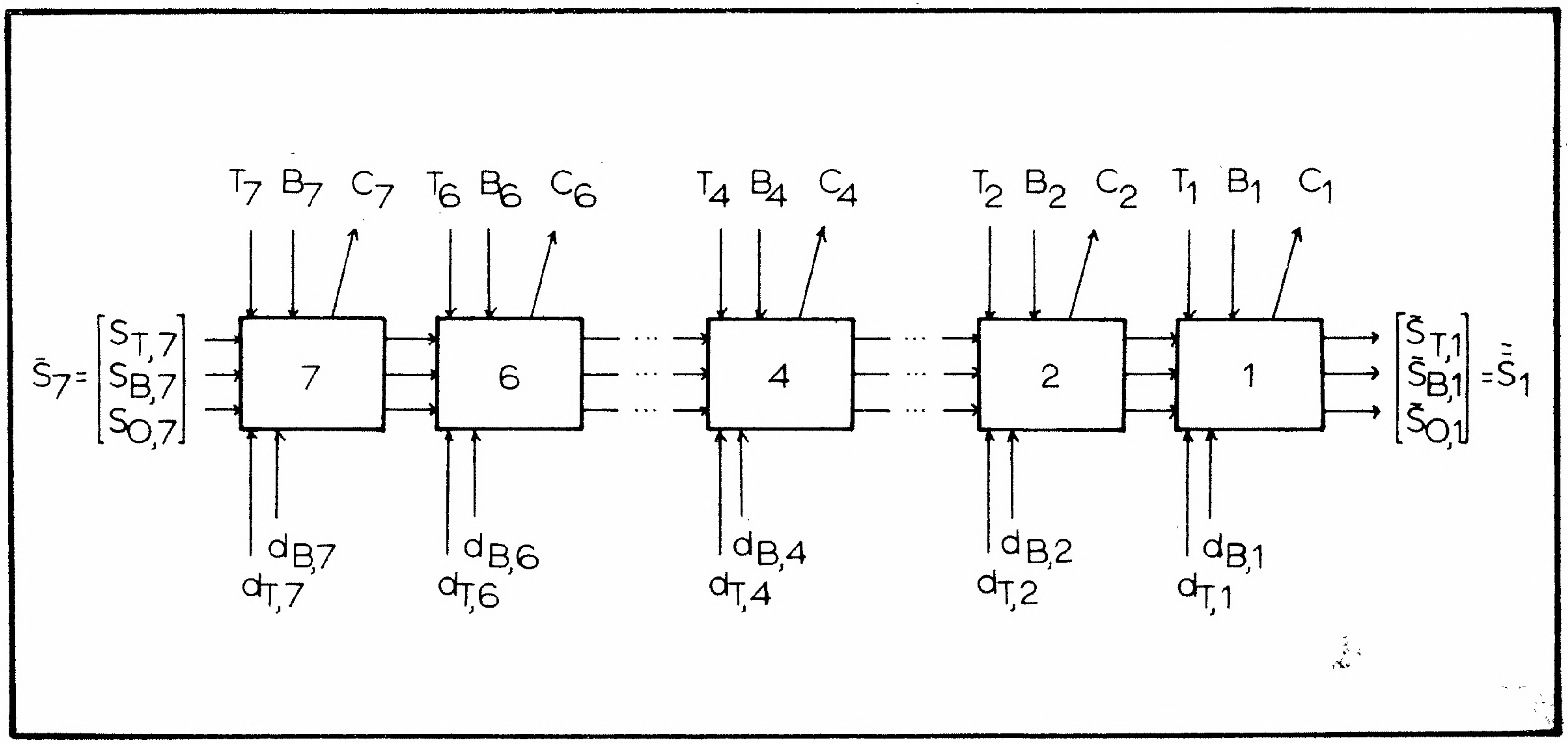

Figure 5. N-Stage Basin Water Quality Model

CHAPTER V

MATHEMATICAL FORMULATION OF THE PROBLEM

Perhaps the first consideration in approaching basin water quality planning and management is the selection of a level or risk. There is a positive probability of violating a standard associated with any abatement program short of complete treatment of all wastes; this is due to the intimate relationship between quality and streamflow. For a given pollution control program, quality is a direct function of flow, which is variable.* It is unrealistic to seek that program which would guarantee a certain water quality or has a probability of violation equal to zero. What must be sought is an abatement program which is economically attractive and has an acceptably low probability of violating a standard.

The usual approach is to specify a design flow, for instance the seven-day, ten-year low flow perhaps for a specific month. This implies a definite probability. An abatement system designed for this flow would be expected to experience a quality violation on the average once every ten years, or during a specific month once every ten years.

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*Low-flow augmentation can increase the mean and reduce the variance of flow for a certain period, thereby reducing the associated risk but not eliminating it. This is similar to the problem of high flows, flood "control"; floods cannot be eliminated (probability of flooding equal zero) but can be "managed"; i.e., the probability of occurrence can be reduced.

With such a design flow, the flow hydrograph, the channel configurations, and time-of-travel information may be determined. Meteorological conditions may be obtained for a similar level of risk. Gross production of thermal and organic wastes may be determined on an annual mean basis or for the particular month being considered.*

Dynamic Programming Approach

This problem can properly be considered as a resource allocation problem. The resource being allocated among competing users is the natural capacity of a stream to assimilate thermal and organic wastes. A portion of this capacity is not available for waste assimilation because of the necessity to protect other users, and this is reflected in the temperature and dissolved oxygen standards.

Using the notation shown in Figure 3, the problem may be formulated as a two-dimensional initial-value dynamic programming problem.**

Given

(a) Water temperature, $S_{T,N}$; biochemical oxygen demand, $S_{B,N}$; and dissolved oxygen, $S_{O,N}$, entering the top of the initial stage, N, of an N-stage process.

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*The use of such annual or monthly mean waste flows is a very severe assumption which must be kept in mind throughout the remainder of this presentation. Waste flows may vary considerably during the year, month, week, or day. The quality of the receiving stream will respond to this variation. Unless otherwise stated, quality standards are instantaneous. Therefore, even though no violation of standards is indicated when using the mean values of waste production in the steady-state situation, violations may in fact occur in the real situation.

**A detailed presentation of the background and theory of dynamic programming is beyond the scope of this investigation. Such information is available in Bellman's work (1,2).

(b) Waste loading, thermal and organic, at each stage, T_n and B_n, respectively, for $n = 1,2,\ldots,N$.

(c) Cost data for cooling and treatment at each stage.

(d) Transfer functions for water temperature, biochemical oxygen demand, and dissolved oxygen for each stage, Φ_n, ψ_n, and Ω_n, respectively, for $n = 1,2,\ldots,N$.

(e) Water quality standards for minimum dissolved oxygen, maximum temperature, and allowable temperature elevation for each stage, DOS_n, TMS_n, and TRS_n, respectively, for $n = 1,2,\ldots,N$,

determine the policy or set of decisions for cooling and treatment levels at each stage such that the total N-stage cost is minimized.

For notational convenience, several vectors will be defined. $\bar{S}_n$ will represent the inputs from the previous stage

$$\bar{S}_n = (S_{T,n}, S_{B,n}, S_{O,n}) \tag{22}$$

Decisions regarding the level of cooling a heated effluent and treating BOD at a particular stage constitute a local abatement policy at stage n. This pair of decisions is designated as $\bar{D}_n$

$$\bar{D}_n = (d_{T,n}, d_{B,n}) \tag{23}$$

The transfer functions, Φ_n, ψ_n, and Ω_n, are used to route temperature, biochemical oxygen demand, and dissolved oxygen through the stage, sub-reach by sub-reach until the end of the stage is reached. For given values T_n and B_n,

$$TEMP_{n,i} = \Phi_n(\bar{S}_n, \bar{D}_n, i) \tag{24}$$

$$BOD_{n,i} = \psi_n(\bar{S}_n, \bar{D}_n, i) \tag{25}$$

$$DIOX_{n,i} = \Omega_n(\bar{S}_n, \bar{D}_n, i) \tag{26}$$

for $i = 0,1,2,\ldots,k_n$ where k_n is the number of sub-reaches in stage n. $TEMP_{n,i}$, $BOD_{n,i}$, and $DIOX_{n,i}$ are the temperature, biochemical oxygen demand, and dissolved oxygen at the end of the ith sub-reach of stage n. This is the input to the next sub-reach, i + 1.

$TEMP_{n,0}$, $BOD_{n,0}$, and $DIOX_{n,0}$ represent mixed quality at the top of stage n; i.e., after the incoming vector is affected by residual heat and organic waste at the nth outfall. $TEMP_{n,k_n}$, BOD_{n,k_n}, and $DIOX_{n,k_n}$ are the values of the quality parameters at the end of the final sub-reach. By convention, the i in the argument of Equations 24, 25, and 26 is deleted when $i = k_n$. Therefore, for given T_n and B_n,

$$\tilde{S}_{T,n} = \Phi_n(\bar{S}_n, \bar{D}_n) \tag{27}$$

$$\tilde{S}_{B,n} = \psi_n(\bar{S}_n, \bar{D}_n) \tag{28}$$

$$\tilde{S}_{O,n} = \Omega_n(\bar{S}_n, \bar{D}_n) \tag{29}$$

The output quality vector from stage n is represented as $\tilde{\tilde{S}}_n$

$$\tilde{\tilde{S}}_n = (\tilde{S}_{T,n}, \tilde{S}_{B,n}, \tilde{S}_{O,n}) \tag{30}$$

At the various discrete points along the stage, the sub-reach boundaries, dissolved oxygen is compared to the standard for a possible violation because it may decrease, increase, or decrease and then increase down the stage. Therefore

$$MINDO_n = Min(DIOX_{n,i}) \tag{31}$$

Since temperature decreases from the top of the stage (i=0) on down to the end,

$$TMAX_n = TEMP_{n,0} \tag{32}$$

a temperature standard violation can be determined from the mixed conditions at the top of the stage by comparing $TMAX_n$ with TMS_n, the maximum allowable temperature in stage n. Also,

$$TRIS_n = (TEMP_{n,0} - S_{T,n}) \tag{33}$$

because the temperature elevation or rise occurs at the outfall or top of the stage.

The set of optimal decisions is represented by $\{d^*_{T,n}, d^*_{B,n}\}$ for $n = 1,2,\ldots,N$, where $d^*_{T,n}$ and $d^*_{B,n}$ are the levels of cooling and treatment (decisions), respectively, at stage n in the final optimal basin policy. The over-all optimal policy is that which minimizes the sum of the N stage costs

$$\text{Min } C = \sum_{n=1}^{N} C_n \tag{34}$$

subject to:

$$MINDO_n \geq DOS_n \tag{35}$$

$$TMAX_n \leq TMS_n \tag{36}$$

$$TRIS_n \leq TRS_n \tag{37}$$

for $n = 1,2,\ldots,N$. $MINDO_n$, $TMAX_n$, and $TRIS_n$ are the minimum dissolved oxygen concentration, maximum temperature, and temperature elevation, respectively, which occur in stage n and are determined from transfer functions Φ_n, ψ_n, and Ω_n as shown previously in Equations 24, 25, 26, 31, 32, and 33. DOS_n, TMS_n, and TRS_n are the corresponding quality standards in stage n.

It follows that, for a particular set of inputs, $\bar{S}_n$, and a particular choice of decisions, $\bar{D}_n$, the vector $\tilde{\bar{S}}_n$ assumes a specific value; and $MINDO_n$, $TMAX_n$, and $TRIS_n$ also assume specific values.

Since a policy that causes a violation of the quality standards or constraints is not admissible, there is no need to evaluate $\tilde{\bar{S}}_n$ for $\bar{D}_n$ such that either

$$MINDO_n < DOS_n \tag{38}$$

$$TMAX_n > TMS_n \tag{39}$$

or

$$TRIS_n > TRS_n \tag{40}$$

Therefore, output values need only be calculated for the $\bar{S}_n$, $\bar{D}_n$ for which the constraints are met. Associated with each $\bar{S}_n$, $\bar{D}_n$ there is a cost

$$C_n = T_n(\bar{S}_n, \bar{D}_n) \tag{41}$$

At stage 1, the problem is as follows. For each vector $\bar{S}_1$, find $\bar{D}_1$ to

$$\text{Min } T_1(\bar{S}_1, \bar{D}_1) \tag{42}$$

Let

$$f_1(\bar{S}_1) = \underset{\bar{D}_1}{\text{Min}}\ T_1(\bar{S}_1, \bar{D}_1) \tag{43}$$

for given values of T_1 and B_1, subject to

$$MINDO_1 \geq DOS_1 \tag{44}$$

$$TMAX_1 \leq TMS_1 \tag{45}$$

$$TRIS_1 \leq TRS_1 \tag{46}$$

Thus for each possible combination of temperature, BOD, and dissolved oxygen coming into the stage, $\bar{S}_1$, there is a substitution of all possible combinations of cooling and treatment ($\bar{D}_1$), and a combination that satisfies the standards (DOS_1, TMS_1, and TRS_1) at the least cost. This $\bar{D}_1$ vector is kept with its cost and output vector, $\tilde{\bar{S}}_1$, for each of the $\bar{S}_1$.

At this point, it may appear that this method is simply a total enumeration process; but Bellman's Principle of Optimality states: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

At stage 2

$$f_2(\bar{S}_2) = \underset{\bar{D}_2}{\mathrm{Min}}\{T_2(\bar{S}_2,\bar{D}_2) + f_1(\Phi_2(\bar{S}_2,\bar{D}_2), \psi_2(\bar{S}_2,\bar{D}_2), \Omega_2(\bar{S}_2,\bar{D}_2)\} \tag{47}$$

for given values of T_j, B_j; $j = 1,2$, subject to

$$MINDO_2 \geq DOS_2 \tag{48}$$

$$TMAX_2 \leq TMS_2 \tag{49}$$

$$TRIS_2 \leq TRS_2 \tag{50}$$

is evaluated for all vectors $\bar{S}_2$.

Continuing to stage n, for each $\bar{S}_n$

$$f_n(\bar{S}_n) = \underset{\bar{D}_n}{\mathrm{Min}}\{T_n(\bar{S}_n,\bar{D}_n) + f_{n-1}\left(\Phi_n(\bar{S}_n,\bar{D}_n),\ \psi_n(\bar{S}_n,\bar{D}_n),\ \Omega_n(\bar{S}_n,\bar{D}_n)\right)\} \tag{51}$$

for given values T_j, B_j; $j = 1,2,\ldots,n$, subject to

$$MINDO_n \geq DOS_n \tag{52}$$

$$TMAX_n \leq TMS_n \tag{53}$$

$$TRIS_n \leq TRS_n \tag{54}$$

is evaluated for each $\bar{S}_n$.

It is noted here that the argument of f_n is a vector with three dimensions. On the left side of the equation, the argument is shown using vector notation; and on the right side, the individual components are used.

At the Nth and last stage

$$f_N(\bar{S}_N) = \underset{\bar{D}_N}{\mathrm{Min}}\{T_N(\bar{S}_N,\bar{D}_N) + f_{N-1}\left(\Phi_N(\bar{S}_N,\bar{D}_N),\ \psi_N(\bar{S}_N,\bar{D}_N),\ \Omega_N(\bar{S}_N,\bar{D}_N)\right)\} \quad (55)$$

for given T_j, B_j; $j = 1,2,\ldots,N$, subject to

$$MINDO_N \geq DOS_N \quad (56)$$

$$TMAX_N \leq TMS_N \quad (57)$$

$$TRIS_N \leq TRS_N \quad (58)$$

is calculated.

However, since $\bar{S}_N$ is known (initial-value formulation), the optimal policy at stage N is readily determined to be $\bar{D}_N^*$, the $\bar{D}_N$ resulting in the least $f_N(\bar{S}_N)$. The output from stage N, $\tilde{\bar{S}}_N$ (and input to stage N-1, $\bar{S}_{N-1}$) is then determined

$$\tilde{S}_{T,N} = \Phi_N(\bar{S}_N,\ \bar{D}_N) = S_{T,N-1} \quad (59)$$

$$\tilde{S}_{B,N} = \psi_N(\bar{S}_N,\ \bar{D}_N) = S_{B,N-1} \quad (60)$$

$$\tilde{S}_{O,N} = \Omega_N(\bar{S}_N,\ \bar{D}_N) = S_{O,N-1} \quad (61)$$

The optimal policy at stage N-1, $\bar{D}^*_{N-1}$, is that $\bar{D}_{N-1}$ corresponding to the minimum $f_{N-1}(\bar{S}_{N-1})$. $\bar{S}_{N-1}$ and $\bar{D}^*_{N-1}$ then allow the determination of $\bar{\bar{S}}_{N-1}$ ($\bar{S}_{N-2}$), etc., on through the entire N stages.

The value of $f_N(\bar{S}_N)$ is the minimum over-all N-stage cost, and the policy associated with it is $[\bar{D}^*_N, \bar{D}^*_{N-1}, \ldots, \bar{D}^*_1]$.

Computational Aspects of the Dynamic Programming Approach

For a typical intermediate stage, n, of the N-stage process, the set of all input vectors, $\bar{S}_n = [S_{T,n}, S_{B,n}, S_{O,n}]$, may be represented as a three-dimensional matrix. Each cell of this "solid" corresponds to a particular $\bar{S}_n$. Associated with each $\bar{S}_n$, is a two-dimensional matrix of local abatement policies; each cell corresponding to a particular $\bar{D}_n = [d_{T,n}, d_{B,n}]$.

For each $\bar{S}_n$, all $\bar{D}_n$ will be considered. Those $\bar{D}_n$ which are feasible, i.e., do not result in a standards violation within the stage, are investigated for cost in the following manner. Given an $\bar{S}_n$ and a feasible $\bar{D}_n$, the output vector, $\bar{\bar{S}}_n$, is determined. This is the input vector to the previous stage, $\bar{S}_{n-1}$. Knowledge of $\bar{S}_{n-1}$ enables one to determine the (previously calculated) (n-1)-stage optimal cost. At the nth stage, the sum of the nth stage local cost and the optimal cost for the (n-1)-stage process* is to be minimized for the specific $\bar{S}_n$ and $\bar{D}_n$. One need go back only to the previous stage, (n-1), due to Bellman's Principle of Optimality. The $\bar{D}_n$ which results in the minimum cost for

*The computational process starts at stage 1, which is the extreme downstream reach. Therefore $\bar{D}^*_1$ is that which minimizes the local 1-stage cost for a particular $\bar{S}_1$.

the entire n-stage process is then the nth component of an n-stage optimal policy for the $\bar{S}_n$.

For each $\bar{S}_n$, the corresponding optimal local policy D_n^* and $\tilde{S}_n$ are stored for future use. Non-optimal $\bar{D}_n$ are discarded.

At the Nth stage (extreme upstream reach), there is only one $\bar{S}_N$ due to the initial-value formulation. The optimal $\bar{D}_N^*$ is determined by minimization of local cost for a $\bar{D}_N$ plus the (N-1)-stage optimal cost associated with the resulting $\tilde{S}_N = \bar{S}_{N-1}$.

The optimal N-stage policy is obtained by passing back through the stages in reverse order. Since there is only one $\bar{S}_N$, $\bar{D}_N^*$ and $\tilde{S}_N$ are known. Since $\tilde{S}_N = \bar{S}_{N-1}$, one may proceed to the appropriate cell in the $\bar{S}_{N-1}$ matrix where $\bar{D}_{N-1}^*$ and $\tilde{S}_{N-1}$ are found. This proceeds in a like manner to stage 1.

CHAPTER VI

ILLUSTRATIVE APPLICATION TO A RIVER BASIN

The Chattahoochee River basin was selected to illustrate the application of the methodology developed in this investigation for several reasons. First, the basin contains significant sources of thermal and organic pollution. Second, federal and state water pollution agencies have conducted water quality investigations on the Chattahoochee; and, therefore, data are available. The third reason for selecting the Chattahoochee River basin is that a federal enforcement conference has been held regarding pollution of interstate waters (the Chattahoochee River) below Atlanta. Several references to the effects of thermal wastes and suspected interaction with organic wastes are contained in the proceedings of this conference (10). Since an earlier enforcement conference in the area (43) had been even more concerned with the interaction of thermal and organic wastes, it was felt that the application of the methodology to the Chattahoochee basin might provide some rational insight to improve planning and management of the basin.

Physical Bounds of the Study

Since there are no significant sources of pollution on the Chattahoochee above Atlanta, Atlanta was selected as the upstream extent of the system.

Hydroelectric power facilities at the Corps of Engineers' Buford Dam, located on the Chattahoochee about 46 miles above Atlanta, are used for peaking power. However, the city of Atlanta has an agreement with the Georgia Power Company to maintain a minimum flow at Atlanta of 750 cubic feet per second (cfs) from its Morgan Falls hydroelectric and reregulation facility located on the Chattahoochee about 12 miles above Atlanta. This tends to damp the diurnal peaking power flows. For the purpose of illustrating the methodology developed in this study, the Chattahoochee River is assumed to be a free-flowing stream. Substantial peaking power flows may very well exist, and this should be kept in mind.

By the time the Chattahoochee reaches the West Point-LaGrange area (about 80-100 miles below Atlanta) all organic and thermal waste effects are sufficiently dissipated to consider this a proper lower bound for the system. The confluence of Yellowjacket Creek with the Chattahoochee at about mile 322* and just above LaGrange will be used.

Organic Waste Sources in the System

The city of Atlanta operates several waste treatment plants that discharge to the Chattahoochee or its tributaries. According to recent operating data (4), 85 per cent of that part of Atlanta's average daily waste flows that drain to the Chattahoochee is discharged at the R. M. Clayton plant (mile 408). The Clayton inflow of 65.5 million gallons

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*Locations along a river are usually described or determined by the distance of the point above the mouth of the river.

per day (MGD) and average BOD of 202 mg/ℓ result in 110,000 pounds of BOD per day. Other published data (10) indicate that approximately 88 per cent of the estimated BOD loading equivalents (PE) before treatment, from Atlanta to just above LaGrange, arrives at the Clayton plant.

On the basis of the above figures, it was concluded that 80 to 90 per cent of the organic waste loading to the system under study would be accounted for by considering only the Clayton plant. The next largest BOD source constituted only 7 per cent of the Atlanta to LaGrange input; the third largest, only 2 per cent.

Thermal Waste Sources in the System

About one mile downstream from the Clayton plant (mile 407) are located two Georgia Power Company steam-electric generating plants, McDonough (598.4 megawatts) and Atkinson (258.0 megawatts). Both use Chattahoochee River flow for condenser cooling water. About 41 miles below McDonough and Atkinson (mile 366) is another Georgia Power Company steam plant, Yates (680.0 megawatts). All of these steam plants can raise the temperature of the river a significant amount at low flows.

In 1966, these three plants, McDonough, Atkinson, and Yates, had annual mean plant factors of .74, .44, and .49, according to a recent Federal Power Commission report (9). At these plant factors, it is estimated that McDonough and Atkinson, combined due to their being adjacent to one another, waste 2.4×10^9 Btu/hr and Yates wastes 1.6×10^9 Btu/hr to the Chattahoochee via their condenser cooling water flows. There are no other significant waste heat sources in the system.

Water Quality Standards in the Chattahoochee River Basin

The Georgia Water Quality Control Board has adopted a set of water use classifications for the Chattahoochee River and has prescribed water quality standards for each use classification (12). The section of the Chattahoochee River being studied in this investigation extends from Atlanta to the LaGrange-West Point area. The use classifications as well as temperature and dissolved oxygen standards for this section are presented in Table 1.

Table 1. Use Classification and Water Quality Standards for Chattahoochee River from Atlanta to the LaGrange-West Point Area

From		To			Standards		
Location	River Mile	Location	River Mile	Use	Dis. Oxygen	Max. Temp.	Max. Temp. Rise
Atlanta	408.1	Cedar Creek	369.1	Industrial	3.0 mg/ℓ	93.2°F	10°F
Cedar Creek	369.1	Franklin	343.1	Fishing	4.0 mg/ℓ	93.2°F	10°F
Franklin	343.1	West Point Dam	309.0	Recreation	4.0 mg/ℓ	93.2°F	10°F

Low-Flow Hydrology of the Chattahoochee River Basin

The low-flow criterion selected for use in this study was the three-day, ten-year low-flow for a particular month at all stations; estimated flow profiles for selected warm-weather months are shown in Figure 6. The selection of the three-day, ten-year criterion was based

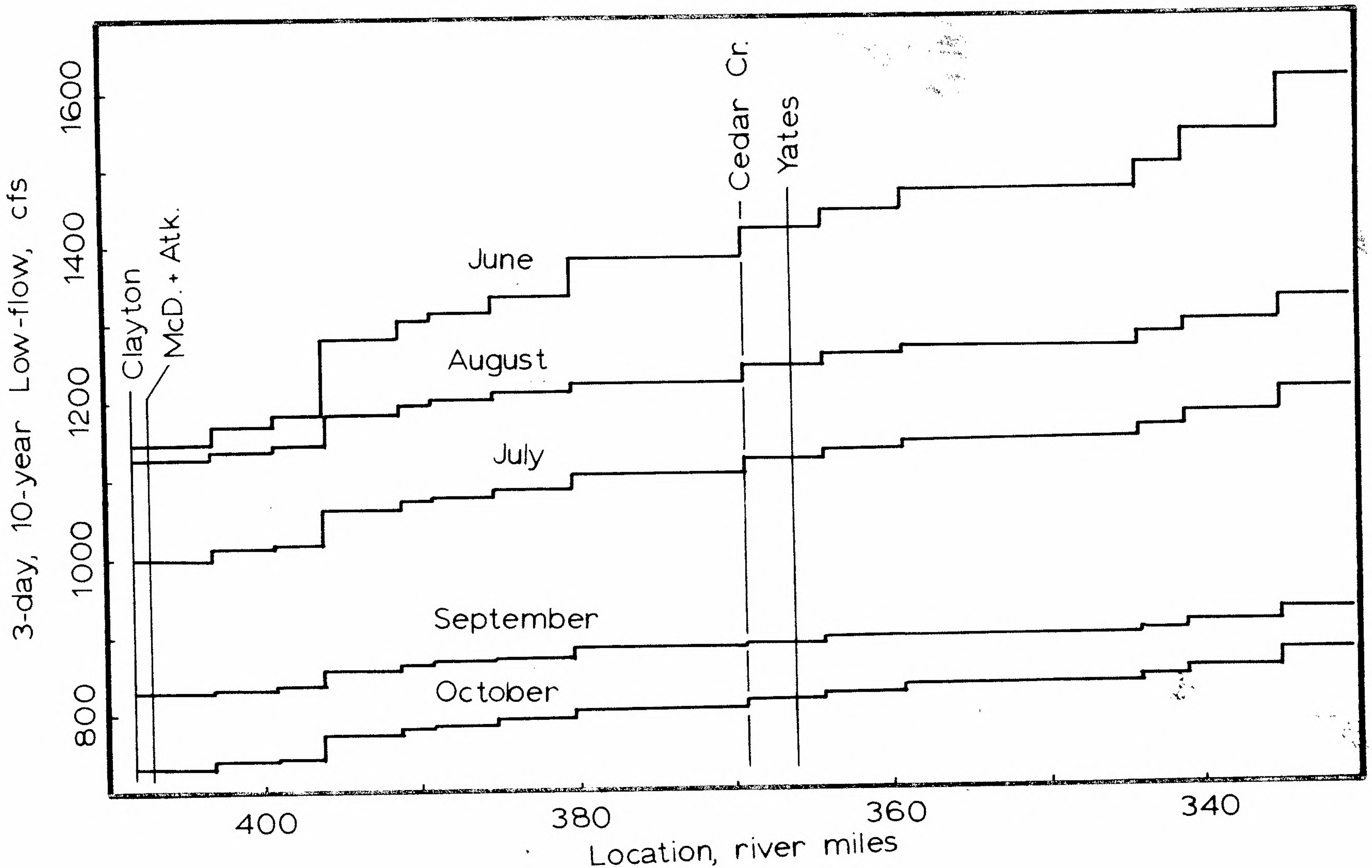

Figure 6. Three-Day, Ten-Year Monthly Low-Flow Profiles for Chattahoochee River Basin

on the availability of such information from a study conducted by a governmental agency. Another criterion, such as the seven-day, twenty-year low-flow, might have been equally appropriate and would have been considered had it been available. For the purpose of illustrating methodology, the three-day, ten-year criterion is satisfactory.

It should be noted in Figure 6 that there are no large tributary inflows during these usually dry months. These profiles were obtained by apportioning the increase in the three-day, ten-year flow (for each month) between gaging stations in proportion to the drainage areas of the intervening tributaries.

Meteorology of the Chattahoochee River Basin

A statistical analysis of meteorological parameters is necessary to obtain values for the determination of the equilibrium water temperatures corresponding to various return periods.

Mean monthly observations* of air temperature, wind velocity, morning and evening relative humidity, solar radiation, and vapor pressure at Atlanta for the period 1950 to 1966 (except for solar radiation, 1951 to 1966) for each month were used to generate curves of mean monthly values. Also the expected value of the adverse once-in-ten-years conditions was determined. These are shown in Figures 7, 8, 9, 10, and 11.

*The incompatability of using mean monthly meteorological data for a system having instantaneous water quality standards is as apparent as it was in the case of annual or monthly mean waste discharges.

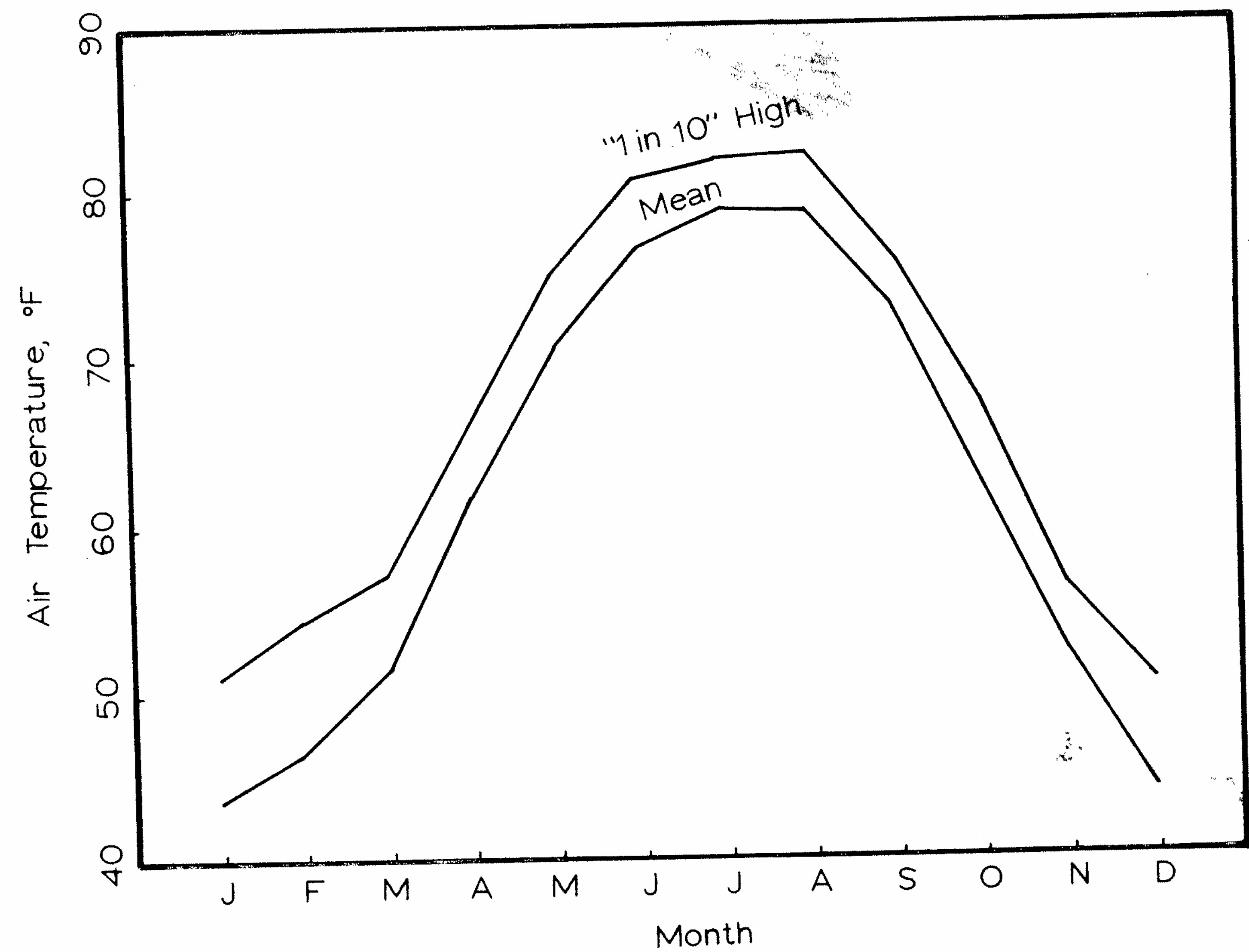

Figure 7. Distribution of Air Temperature

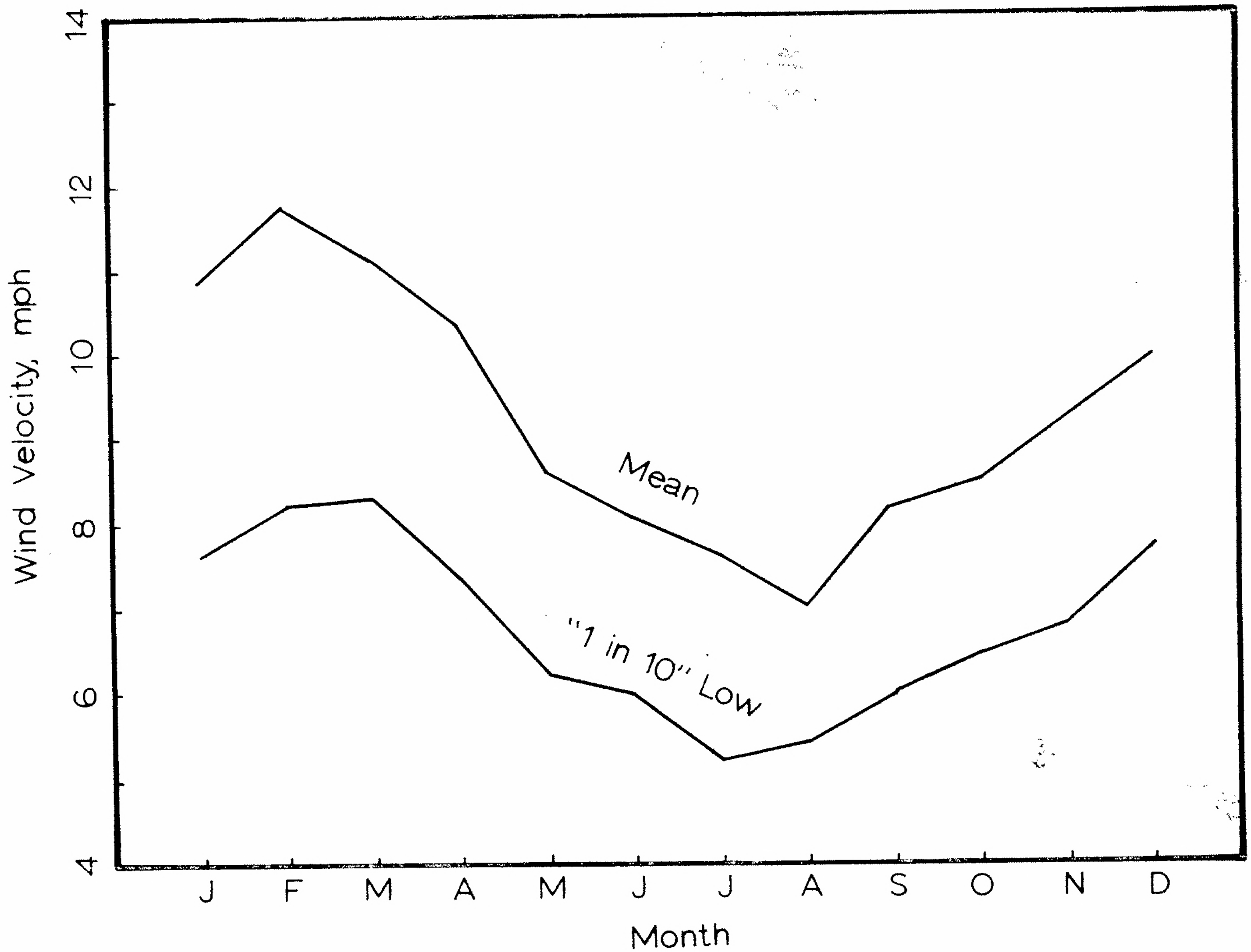

Figure 8. Distribution of Wind Velocity

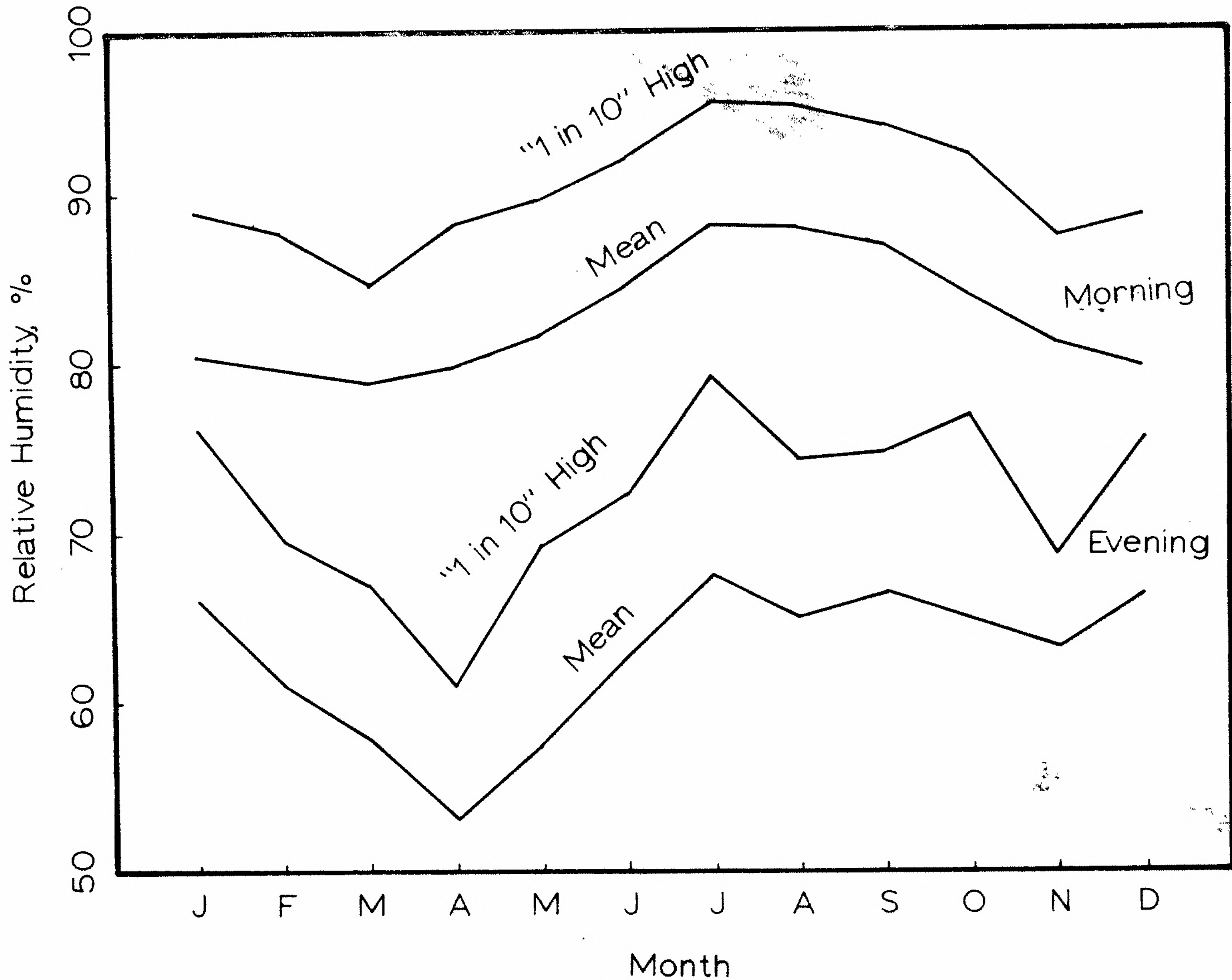

Figure 9. Distribution of Morning and Evening Relative Humidity

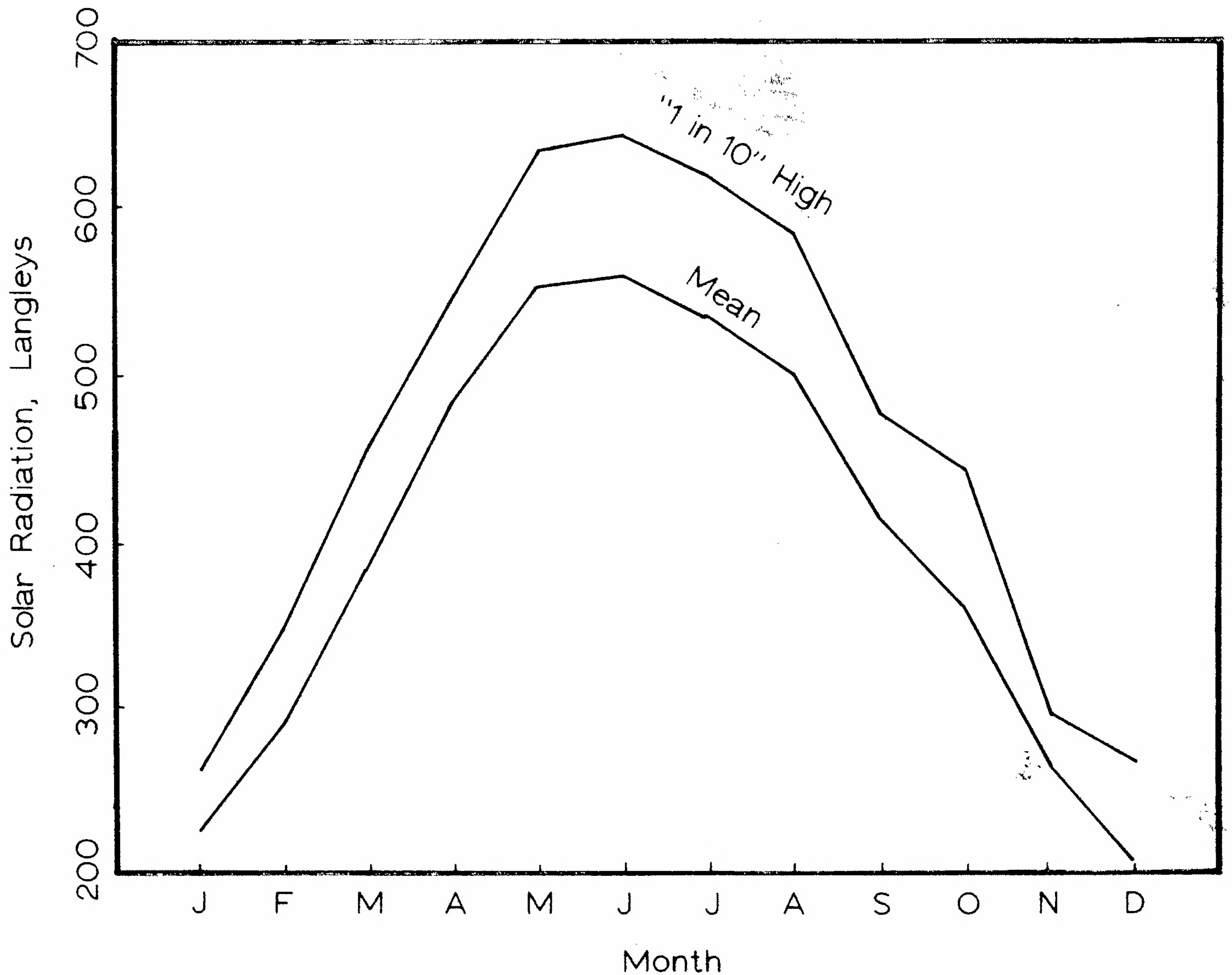

Figure 10. Distribution of Solar Radiation

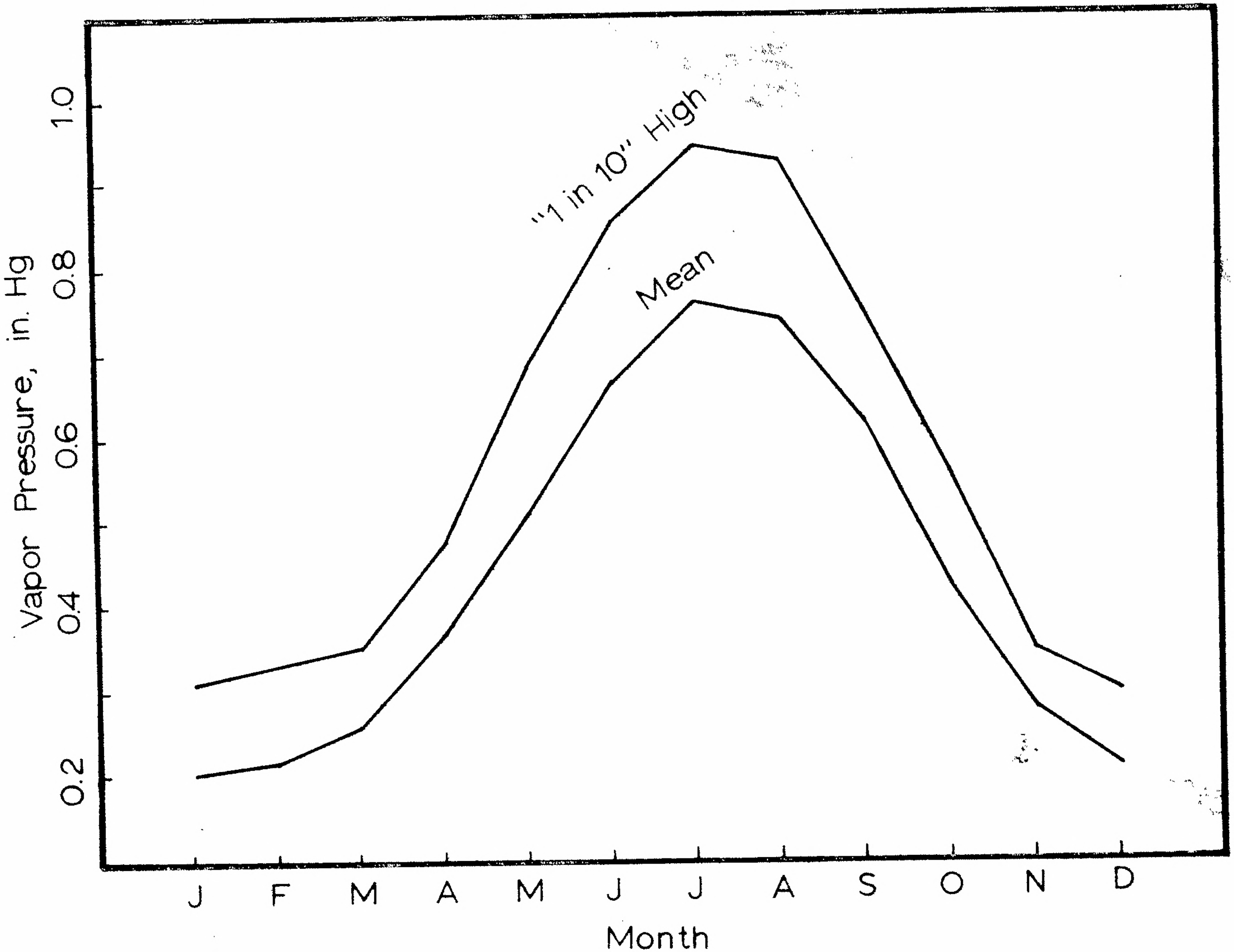

Figure 11. Distribution of Vapor Pressure

The methodology of Velz and Gannon (44) was used to select appropriate values of the meteorological parameters and to generate annual equilibrium water temperature curves.* These are shown in Figure 12.

It was felt that Atlanta meteorology adequately characterized that of the system being studied. The once-in-ten-years adverse meteorological conditions and the once-in-ten-years high equilibrium water temperature, as determined by the method of Velz and Gannon (44), were selected for use in this study.

It is of interest to consider actual data on air and water temperature collected at the Atlanta Water Works intake on the Chattahoochee River during July and August of 1968. The July and August mean air temperatures were 78.4 and 79.7°F, respectively. These values are within about a degree of the mean air temperature curve shown in Figure 7. The mean water temperatures for July and August were 61.4 and 60.2°F, respectively; about 20° below the mean equilibrium water temperature curve shown in Figure 12.

These data, limited as they are, cast considerable doubt on the validity of the Velz-Gannon equilibrium water temperature model as used for the investigation of the Chattahoochee River basin. Whenever a significant discrepancy is apparent between the results of such a mathematical model and reliable, historical data, further study should be undertaken to resolve the question.

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*This will tend to give a conservatively high value for the one-in-ten adverse conditions as it is unlikely that all meteorological parameters would be this at the same time.

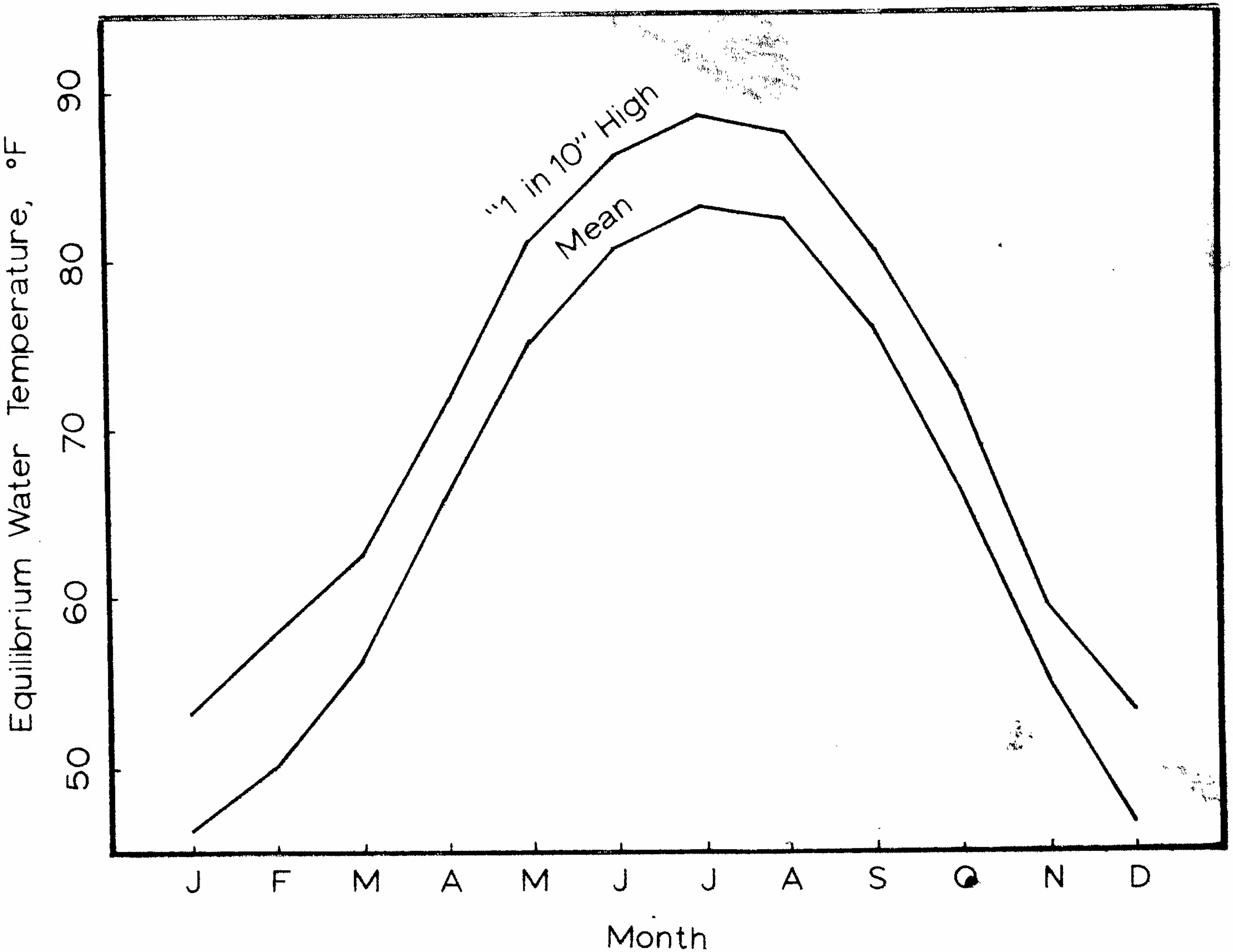


Figure 12. Distribution of Equilibrium Water Temperature

One would expect water discharged from Buford Dam during the warm months to be cooler than air temperature and probably cooler than the equilibrium water temperature. During the time required for the water to travel the 46 miles from Buford Dam to Atlanta, the water temperature should increase to a value which is close to the equilibrium temperature so that the cool releases from Buford should not be a significant factor in the water temperature below Atlanta.

If the actual equilibrium water temperature of the Chattahoochee River basin is indeed much lower than that used in the subsequent illustration, the effect would be to make the maximum water temperature standard (93°F) completely inoperative as a constraint. The reason for this is that the maximum water temperature elevation standard (10°F) would prevent water temperature from getting near the 93°F point.

Division of System into Stages

The upper and lower bounds of the system have been determined as Clayton plant (mile 408) and Yellowjacket Creek (mile 322), respectively. Intermediate boundaries are indicated for significant waste sources between these limits, McDonough and Atkinson steam plants (mile 407) and Yates steam plant (mile 366).

From Table 1, it is noted that maximum temperature and temperature elevation standards are constant throughout the system. Dissolved oxygen standards increase from 3 to 4 mg/ℓ at Cedar Creek (mile 369); therefore, an intermediate boundary is indicated at mile 369.

It is seen in Figure 6 that no intermediate boundaries are required due to significant tributary inflows.

Figure 13 shows a map of the section of the Chattahoochee River basin that constitutes the system with the above extreme and intermediate boundaries delineated. It is felt that four stages (N = 4) will adequately characterize the system for water quality planning purposes. The system is shown as an N-stage process in Figure 14.

Heat Dissipation in the Chattahoochee River Basin System

Given the appropriate meteorological conditions, equilibrium water temperature, and flow profile, one may readily determine the temperature profile induced by 1°F elevation (over equilibrium) of water temperatures at the top of each stage. Discrete points on these profiles are referred to subsequently as "unit ordinates."

The actual temperature profile down a stage is obtained by scaling these unit ordinates by the temperature elevation that results from a thermal influx. This multiplier, $FORCE_n$, is obtained as follows

$$FORCE_n = S_{T,n} + \frac{4.45 \times 10^{-8} \, (100 - d_{T,n}) T_n}{Q_n} - TEQ \qquad (62)$$

in which

$FORCE_n$ = Temperature elevation above equilibrium at outfall n, °F

$S_{T,n}$ = Temperature of water entering stage n, °F

$d_{T,n}$ = Level of cooling at n, percentage

T_n = Waste heat produced at n, Btu/hr

Q_n = Mean stream flow in stage n, cfs

TEQ = Equilibrium water temperature, °F.

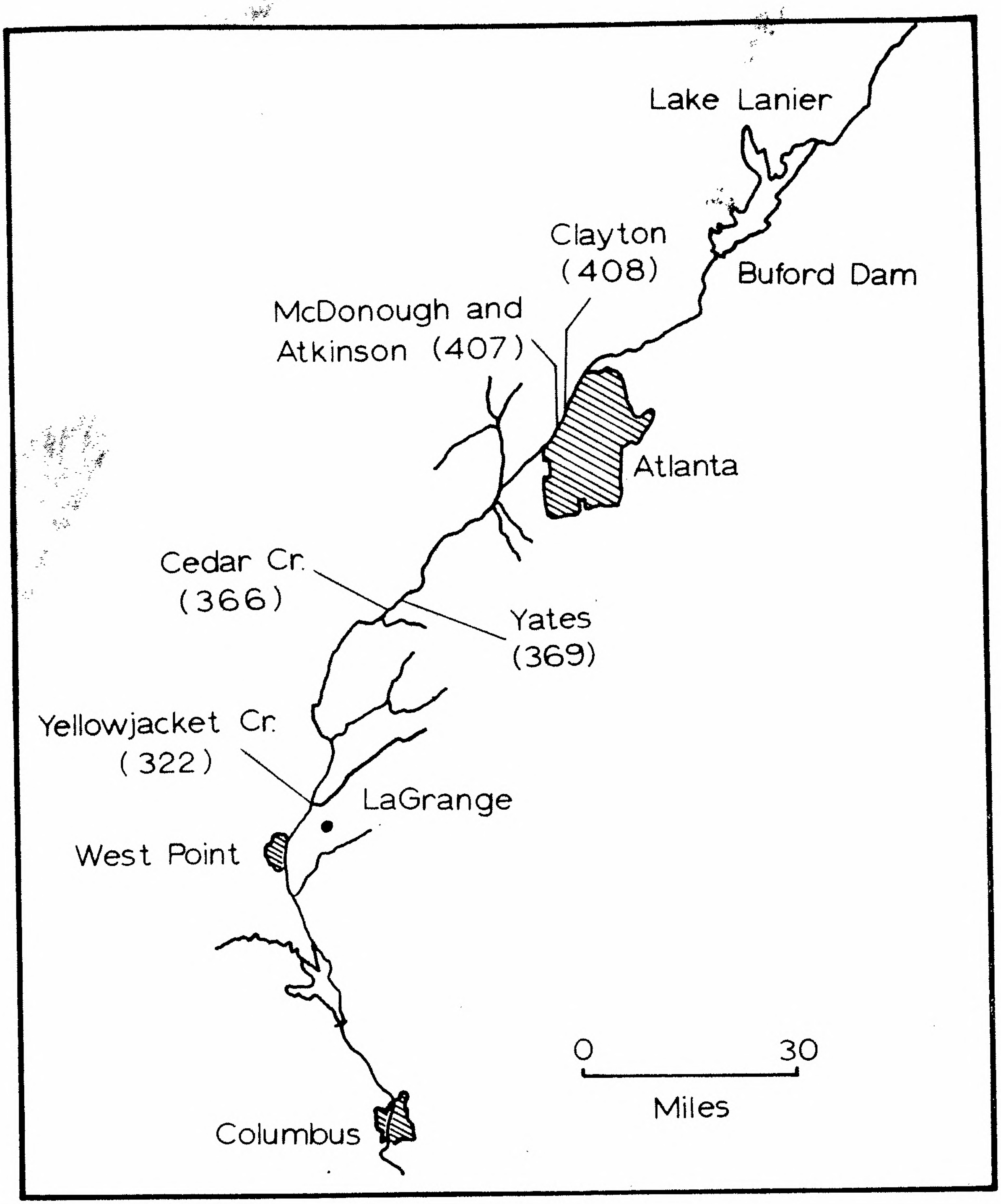
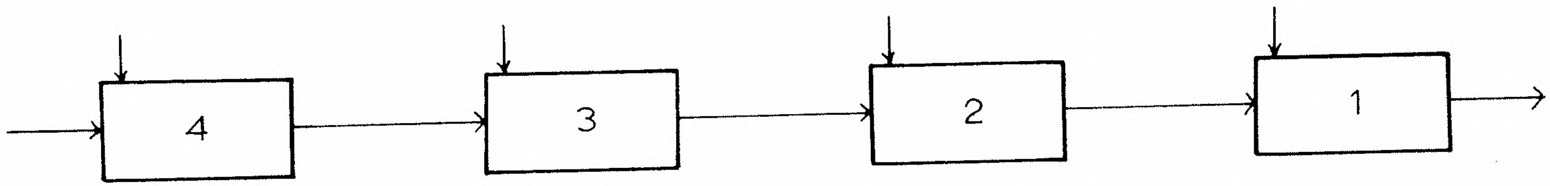


Figure 13. Section of Chattahoochee River Basin Constituting System



Stage	Location	Waste	Water Quality Standards		
			DO(mg/ℓ)	TMAX(°F)	TRIS(°F)
4	Clayton to McDonough and Atkinson (river mile 408-407)	organic	3	93	10
3	McDonough and Atkinson to Cedar Creek (river mile 407-369)	thermal	3	93	10
2	Cedar Creek to Yates (river mile 369-366)	none	4	93	10
1	Yates to Yellowjacket Creek (river mile 366-322)	thermal	4	93	10

Figure 14. Chattahoochee River Section Shown as N-Stage Process

Rate Coefficients and Time-of-Flow Used in the Chattahoochee River Basin System

The importance of the deoxygenation and reaeration coefficients in any water quality investigation is obvious. It is unfortunate that these coefficients are often difficult to determine. The following information regarding rate coefficients and velocities of streamflow were obtained by the author from a governmental agency which has conducted studies of the Chattahoochee River. The relationships are based on very limited survey data of a preliminary nature. It is felt that the relationships are adequate for illustrative purposes.

The 20°C deoxygenation rate coefficient, K_1, used in this investigation is 1.0/day.* It is known that K_1 is a function of the level of treatment. The 20°C reaeration rate coefficient, K_2, was found to be related to the rate of streamflow as follows

$$K_2 = 46\ Q_n^{-0.566} \tag{63}$$

where Q_n is the mean flow (cfs) in stage n. A value of 0.9/day is thought to be applicable from Atlanta to West Point at low-flow conditions.

From Atlanta to Whitesburg (about mile 368) the following equation relates mean velocity (fps) to low-flow (cfs)

$$V_n = 0.0685\ Q_n^{0.428} \tag{64}$$

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*All rate coefficients are base "e."

From Whitesburg to West Point, the following equation is applicable

$$V_n = 0.0515\ Q_n^{0.451} \tag{65}$$

Time-of-flow in days, TF_n, in a stage is determined by the following

$$TF_n = \frac{Q_n}{16.4\ V_n} \tag{66}$$

where the constant, 16.4, converts velocity in fps to mi/day. The total time-of-flow in a stage is divided into segments corresponding to the flow-times between the discrete unit ordinates described under the temperature dissipation section.

Initial Conditions

As this system has been formulated as an initial-value dynamic programming problem, the initial state vector, $\bar{S}_N$, must be evaluated.

It is noted from Figure 12 that the September once-in-ten-years high equilibrium temperature is 80.7°F. For the purposes of this investigation, it has been assumed that the BOD concentration present in the Chattahoochee River above Atlanta is 2 mg/ℓ which represents diffuse, low-level organic pollution. It is further assumed that the dissolved oxygen concentration of the Chattahoochee River at Atlanta is about 85 per cent of saturation during low-flow periods. Therefore, 6.7 mg/ℓ DO is used.

The resulting initial conditions are then

$$\bar{S}_4 = \begin{bmatrix} S_{T,4} \\ S_{B,4} \\ S_{O,4} \end{bmatrix} = \begin{bmatrix} 80.7\ ^{\circ}F \\ 2.0\ mg/\ell \\ 6.7\ mg/\ell \end{bmatrix} \tag{67}$$

Limits on Inputs and Abatement Measures

For each stage in the system, one must establish upper and lower limits on incoming water temperature, BOD, and DO, as well as levels of cooling of thermal wastes and treatment* of organic wastes.

Where thermal and/or organic wastes are produced, limits on these abatement measures should be allowed to vary from 0 to 100 per cent. If there is no thermal waste or no organic waste or neither, the appropriate limits would be 0 to 0 per cent.

Given the initial conditions entering the system ($\bar{S}_N$), streamflow, meteorology, rate coefficients, water quality standards, and production of thermal and organic wastes throughout the system, one may determine the upper bounds on the state variables entering each stage; i.e., water temperature, BOD, and DO, as follows.

Water Temperature

First determine the maximum possible elevation of stream temperature due to each heat source using Equation 68. This is done by

*Though cooling is a type of waste treatment, the term "treatment" will be used subsequently to refer exclusively to the abatement of conventional oxygen-demanding organic wastes (BOD).

setting the level of cooling equal to 0 per cent. For-once-in-ten-years September low flow and adverse meteorology in the Chattahoochee River basin, this will be determined for the two heat sources, Plants McDonough and Atkinson (combined) and Plant Yates, using the following

$$TRIS_n = \frac{4.45 \times 10^{-8} \; (100 - d_{T,n})T_n}{Q_n} \qquad (68)$$

in which

$TRIS_n$ = Water temperature elevation at the top of stage n, °F.

$d_{T,n}$ = Level of cooling at stage n, percentage.

T_n = Heat wasted to cooling water at stage n, Btu/hr.

Q_n = Mean flow in stage n, cfs.

Setting $d_{T,n}$ equal to 0, Equation 68 reduces to

$$MAXTRIS_n = \frac{4.45 \times 10^{-6} \; T_n}{Q_n} \qquad (69)$$

where $MAXTRIS_n$ is the maximum possible mixed temperature rise at n in °F.

Results of Equation 69 as well as information required for its evaluation are shown in Table 2.

One may now start at the upstream end of the system and route the maximum heat conditions through the system, stage by stage. All that is needed are the initial entering water temperature ($S_{T,N}$), the $MAXTRIS_n$'s, and the terminal unit temperature ordinated ($TORD_{n,i\text{-}final}$) for each stage. ST_N is 80.7°F; the $MAXTRIS_n$'s are shown in Table 2

Table 2. Maximum Temperature Elevations Due to Heat Sources in Chattahoochee River Basin for One-in-Ten September Conditions

Stage, n	Source	River Mile	T_n (Btu/hr)	Q_n (cfs)	$MAXTRIS_n$ (°F)
4	Clayton	408	0	820	0.0
3	McDonough and Atkinson	407	2.4×10^9	870	12.3
2	Cedar Creek	369	0	885	0.0
1	Yates	366	1.6×10^9	920	7.7

as 0.0, 12.3, 0.0, and 7.7°F for n = 4,3,2,1. The $TORD_{n,i\text{-final}}$ are 0.94, 0.09, 0.83, and 0.04 for n = 4,3,2,1. This means, for example, that 9 per cent of the initial elevation above the equilibrium water temperature at stage 3 (McDonough and Atkinson) will remain when stage 2 is reached.

Since no heat is added in stage 4, water temperature remains at 80.7°F to stage 3, where a 12.3°F rise may occur. The initial elevation at stage 3 of 12.3°F is multiplied by $TORD_{3,i\text{-final}}$ (0.09) to obtain the elevation above equilibrium at the end of stage 3 and entering stage 2. This is 1.1°F and yields a water temperature coming into stage 2 of 81.8°F (80.7 + 1.1). Since there is no additional elevation at stage 2, the final elevation out of stage 2 and into stage 1 is 1.1°F multiplied by $TORD_{2,i\text{-final}}$ (0.83) or 0.9°F, yielding a water temperature of 81.6°F (80.7 + 0.9). At stage 1, an additional elevation of 7.7°F may occur. The initial elevation of 0.9°F plus the 7.7°F results in a total elevation at the top of stage 1 of 8.6°F which, when scaled by $TORD_{1,i\text{-}final}$ (0.04), gives a water temperature out of stage 1 and out of the system of 81.0°F (80.7 + 0.3). This completes the maximum temperature profile. The minimum possible temperature profile would result from total cooling of thermal wastes and would simply be a horizontal line at the equilibrium temperature. The limits on $S_{T,n}$ are presented in Table 3.

Table 3. Range of Incoming Water Temperatures

Stage, n	River Mile	$S_{T,n}$, °F	
		min	max
4	408	80.7	80.7
3	407	80.7	80.7
2	369	80.7	81.8
1	366	80.7	81.6

Biochemical Oxygen Demand

Calculation of BOD is as described below. The maximum possible increase in the BOD concentration due to organic waste discharges throughout the basin is determined by using the following equation where the level of treatment is equal to 0

$$BODINCR_n = \frac{0.1855\ B_n}{Q_n} \tag{70}$$

in which

$BODINCR_n$ = Increase in BOD concentration due to discharge at n, mg/ℓ.

B_n = Organic waste produced at n, #/day.

Q_n = Mean flow in stage n, cfs.

Results of Equation 70 and data required for its evaluation are shown in Table 4. The minimum value of $BODINCR_n$ would be zero for all stages, corresponding to complete treatment.

Table 4. Max BOD Increases Due to Organic Waste Sources in Chattahoochee River Basin for One-in-Ten September Conditions

Stage, n	Source	River Mile	B_n (#/day)	Q_n (cfs)	$BODINCR_n$ (mg/ℓ)
4	Clayton	408	110,000	820	20.4
3	McDonough & Atkinson	407	0	870	0.0
2	Cedar Creek	369	0	885	0.0
1	Yates	366	0	920	0.0

One may determine the maximum and minimum BOD profile through the system by selecting appropriate values of the deoxygenation rate coefficient, K_1, and routing the BOD through the system, stage-by-stage. For the maximum profile, the 20°C K_1 (1.0) is corrected to the equilibrium temperature (80.7°F), giving a K_1 value of 1.4/day. For the minimum profile, K_1 is corrected to correspond to the maximum allowable water temperature, in this case 93°F, giving a value of 1.9/day. The 1.4/day will give slow dissipation of BOD, and the 1.9/day, a more rapid dissipation. These two conditions correspond to uniformly low and high water temperature, respectively, and include all possible BOD profiles.

The profiles are obtained using the following equation

$$BOD_n = (BOD_{n+1} + BODINCR_n) \exp(-K_1 \cdot TF_n) \qquad (71)$$

in which

BOD_n = BOD concentration at the end of n, mg/ℓ.

BOD_{n+1} = BOD concentration at the end of the previous stage, n + 1, which enters stage i, mg/ℓ.

$BODINCR_n$ = Increase in BOD due to discharge at n, mg/ℓ.

K_1 = Deoxygenation rate coefficient, 1/days.

TF_n = Time-of-flow through stage n, days.

Results of the use of Equation 71 and data used are shown in Table 5. The appropriate limits on SB_n are presented in Table 6.

Table 5. Summary of Calculations for Maximum and Minimum BOD Profiles for Chattahoochee River Basin for One-in-Ten Adverse September Conditions

Stage, n	River Mile	TF_n Days	Minimum K_1 (1/days)	Minimum BOD_{n+1} (mg/ℓ)	Minimum $BODINCR_n$ (mg/ℓ)	Minimum BOD_n (mg/ℓ)	Maximum K_1 (1/days)	Maximum BOD_{n+1} (mg/ℓ)	Maximum BODINCR (mg/ℓ)	Maximum BOD_n (mg/ℓ)
4	408	0.05	1.9	2.00	0.0	1.82	1.4	2.00	20.4	20.90
3	407	1.87	1.9	1.82	0.0	0.05	1.4	20.90	0.0	1.52
2	369	0.17	1.9	0.05	0.0	0.04	1.4	1.52	0.0	1.20
1	366	2.40	1.9	0.04	0.0	0.00	1.4	1.20	0.0	0.04

Table 6. Range of Incoming BOD

Stage, n	River Mile	$S_{B,n}$, mg/ℓ min	max
4	408	2.00	2.00
3	407	1.82	20.90
2	369	0.05	1.51
1	366	0.04	1.20

Dissolved Oxygen

The lower limit on DO entering a stage may be taken as the DO standard pertaining to that stage. Any value less than this will be unacceptable since a violation of water quality standards would automatically occur. For the upper limit on DO throughout the system, one may select the DO saturation value corresponding to the equilibrium temperature.* The Nth stage is a special case in that incoming DO is specified.

For the once-in-ten-years adverse September conditions, the appropriate limits on DO entering the four stages ($S_{0,n}$) in the Chattahoochee basin system will be developed as an illustration. The value of $S_{0,4}$ was determined previously to be 6.7 mg/ℓ (.85 x 7.9 mg/ℓ). For convenience, 8.0 mg/ℓ is used as the upper limit throughout the system, though 7.9 mg/ℓ is the saturation DO concentration corresponding

*When supersaturation is a possibility, this must be considered.

to the equilibrium temperature of 80.7°F. The DO standards for the stages are 3.0, 3.0, 4.0, and 4.0 mg/ℓ for n = 4,3,2,1, respectively. The DO limit information is presented in Table 7. Any acceptable abatement schedule will provide DO entering the stages within these limits.

Table 7. Range of Incoming Dissolved Oxygen

Stage, n	River Mile	$S_{0,n}$, mg/ℓ	
		min	max
4	408	6.7	6.7
3	407	3.0	8.0
2	369	4.0	8.0
1	366	4.0	8.0

<u>Increments on Abatement Levels and Inputs</u>

The size of increments used for percentage cooling and treatment, and incoming water temperature, BOD, and DO at each reach will determine the density of points that will be investigated on the total cost response surface. The results of this investigation indicate that, for a given set of conditions, there is a unique point of optimality, though the region around it is rather "flat." The finer the increments on abatement and inputs, the more points there will be around the point of optimality, and, hence, the closer one may get to it, though the improvement in the objective function decreases rapidly as the point of optimality is approached.

One constraint on increment size is computer capability. As one uses smaller increments, the core storage and run-time increases geometrically. A second constraint is that, below a certain point, smaller increment sizes have little physical significance. For example, the utility of increments on cooling and treatment being below 1 or 2 per cent is doubtful; if a treatment facility were specified in the optimal schedule to be 32 or 37 per cent, primary treatment at about 35 per cent would probably be used. It is thought that processes cannot be operated to provide reductions with such precision. Also, when one uses very small increments on temperature, BOD, and DO, say 0.10°F or 0.25 mg/ℓ, this may be more precise than the parameters can be measured or are known.

A rational compromise in the selection of increment sizes is indicated between precision and utility. An alternative is to initiate the investigation with rather coarse increments,* eliminate regions of the response surface that are clearly distant from the point of optimality, and use successively smaller increments to investigate the reduced region. In this manner, one will probably approach the optimal as closely as desired. This is essentially the approach suggested by Liebman (21) and is efficient with regard to computer time. An option to the basic computer program developed in this investigation does this automatically.

*For example, 5 per cent on cooling and treatment, 1/2°F on temperature, 2 mg/ℓ on BOD, and 1/2 mg/ℓ on DO.

Development of Cost Data

Cost data must be available or must be developed that relate annual cost of abatement for each facility to per cent abatement. It is very difficult to obtain reliable, generally accepted cost data. For the Chattahoochee River basin, cost data did not exist and was, therefore, calculated using the best available published information.

Treatment Costs

The only significant source of organic waste in the Chattahoochee basin is, as discussed previously, Atlanta's R. M. Clayton treatment plant which receives a mean flow of 65.5 MGD. The only cost data available from the City of Atlanta was for current operating costs for primary treatment. Frankel's 1965 data (11) for 2.5, 10, and 50 MGD plants was extrapolated to 65.5 MGD. Annual cost figures for percentage treatment through 95 per cent were taken from the resulting graph. For 100 per cent treatment, distillation cost (13) was used. A linear relationship was used between 95 and 100 per cent. The dearth of cost information, questionable or reliable, necessitated this approach. It is felt that the data developed is adequate for the demonstration of the methodology developed in this investigation.

The resulting cost versus percentage abatement relationship for the Clayton plant is shown in Figure 15. Since the results of an optimization process will be very strongly influenced by the cost relationships used, it is appropriate to comment on the shape of the cost curve presented in Figure 15.

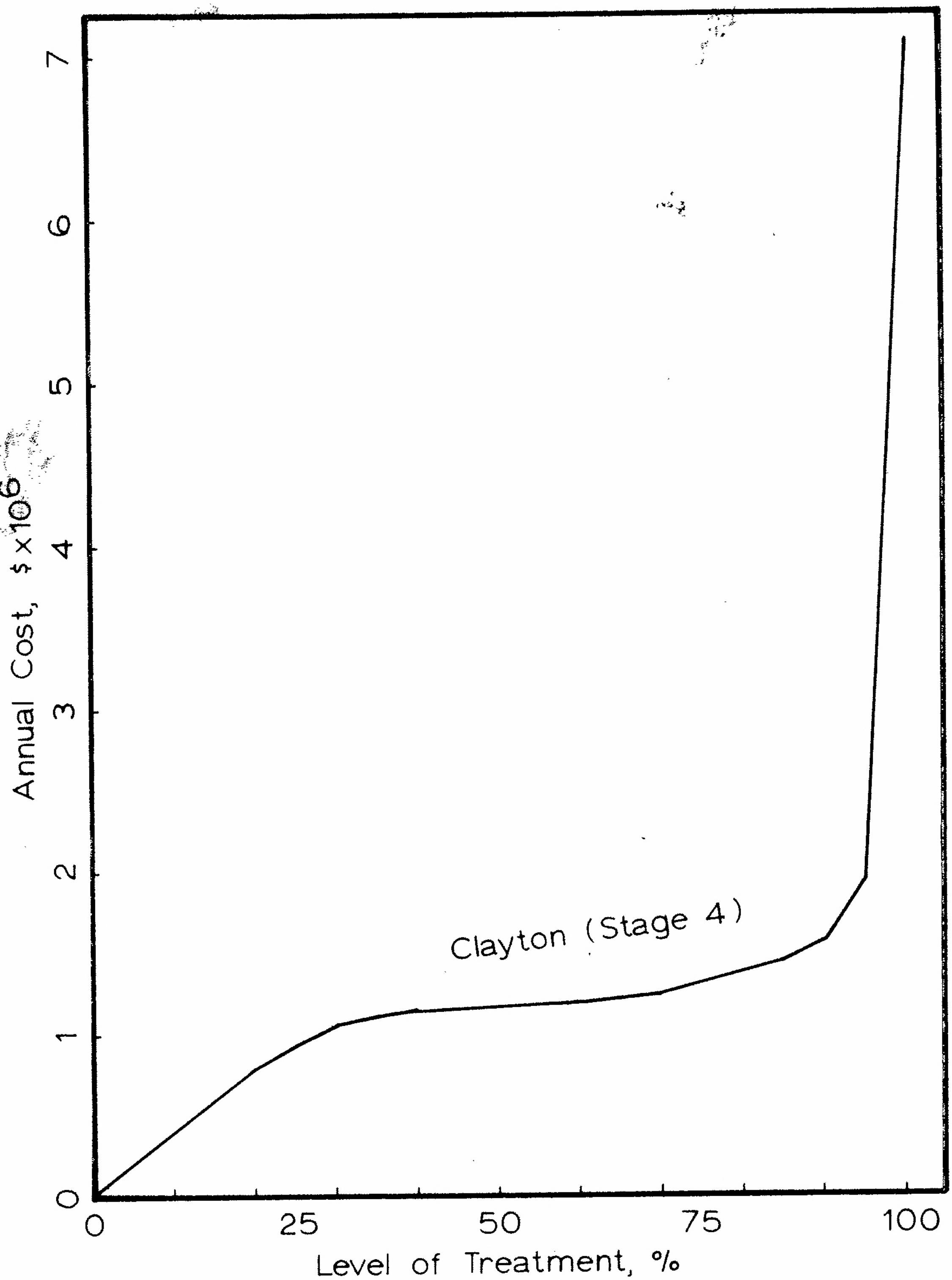

Figure 15. Annual Cost versus Abatement Level, Clayton Plant

One would anticipate a rather uniform increase in cost (and rather constant marginal cost) from a point corresponding to primary treatment (about 35 per cent) up to the lower limit of secondary treatment (about 70 per cent). In a particular case the slope of the curve in this region could be as low as that shown in Figure 15 or much higher. As one reaches the lower limit of secondary treatment, a sharper break upward to a second region of rather uniform cost increase might be expected due to the need for more expensive treatment processes. This might be in the form of a step increase or a ramp increase. The existence of such an increase and its nature should be determined from a detailed consideration of the alternative treatment processes applicable to a specific wastewater treatment facility. Any variation of the shape of a facility's cost curve would affect the rate of substitution of abatement at that facility with other abatement facilities in the basin.

Cooling Costs

The system's two significant heat sources, Plants McDonough and Atkinson (combined) and Plant Yates, presented an even greater problem with regard to abatement cost information. Numerous articles on cooling facilities were consulted. These were of little assistance in the current investigation as they were specific to a certain type of facility or a case study of an installation. The work of Cootner and Löf (6) was selected as the most general and definitive work on the subject of cooling costs. Cootner and Löf present methodology for determining the capital and operating costs of cooling facilities on a cents/1000 gallon circulated basis.

An adaptation of Cootner and Löf's formulation provides the capital cost of a cooling tower

$$\text{CAP COST} = 8KQ\left[\frac{r + 0.03 + t}{5.256\ n \cdot Q \cdot Pf}\right] \tag{72}$$

in which

CAP COST = Capital cost, cents/1000 gallons circulated.

8 = Average capital cost, \$/gpm.

K = Relative rating factor, which is a function of the range, approach, and wet-bulb temperature.

Q = Condenser cooling flow, gpm.

r = Cost of capital, decimal.

t = Property tax rate, decimal.

n = Proportion of year cooling facility used, decimal.

Pf = Plant factor, decimal.

The figure of \$8/gpm agrees with that of a power industry source consulted. The sum of r and t is assumed to be 0.07. The value of n used is 0.25, reflecting the anticipated use of the facility for approximately three months out of the year.

Operating costs are given by the following relationship

$$\text{OP COST} = 0.00113\ R(6.9 + Wa) + (0.14\ K + 0.005A)p \tag{73}$$

in which

OP COST = Operating cost cents/1000 gallons circulated.

R = Range, °F.

Wa = Acquisition cost of cooling water, cents/1000 gal.

K = Relative rating factor.

A = Pumping head, ft.

p = Power costs, cents/kwh.

It is assumed that the acquisition cost of water (Wa) is zero, since it is being taken from the river. Though quite variable due to terrain, the pumping head (A) is assumed to be 70 feet; and the power cost (p) used is 3 mills/kwh.

Data generated and 100 per cent cooling costs for the Chattahoochee River heat sources are presented in Table 8.

Since annual cost must be known as a function of the level of cooling, exponential relationships were developed for the two heat sources, with the 100 per cent cooling cost being that calculated. The parameters of the cooling cost models were selected to assure a constantly decreasing marginal cost and to incur about 75 per cent of the 100 per cent cooling cost at 50 per cent cooling. Due again to the absence of reliable information, the above approach is thought to be a reasonable approximation of the cost functions.

The form of the cooling cost model used is

$$\text{COOL COST}_{n,M} = \frac{\text{COOL COST}_{n,100}}{0.9}\,[1 - \exp(-0.023\ M)] \tag{74}$$

Table 8. Data and Resulting Cooling Costs for Steam Electric Generating Plants in Chattahoochee River Basin for One-in-Ten Adverse September Conditions

Heat Source	Approach (°F)	Range (°F)	K	Q (gpm)	r+t	n	Pf	Capital Cost ¢/10^3gal	Wa ¢/10^3gal	A(head) (ft.)	Power Cost(p) (¢/kwh)	Operating Cost ¢/10^3gal	Total Cost ¢/10^3gal	Annual Cost ($/yr)
McDonough	9.2	18	1.6	273,000	0.07	0.25	0.74	1.32	0	70	0.3	0.31	1.63	434,000 *
Atkinson	9.2	13	1.4	300,000	0.07	0.25	0.44	1.94	0	70	0.3	0.26	2.20	383,000
Yates	9.2	16	1.5	411,200	0.07	0.25	0.49	1.89	0	70	0.3	0.29	2.16	575,000

*McDonough and Atkinson combined is $817,000.

in which

COOL COST = Annual cooling cost for stage n at M% cooling \$.

$\text{COOL COST}_{n,M}$ = Annual cooling cost for stage n at 100% cooling, \$.

M = Level of cooling at stage n*, %.

The resulting cooling cost curves for stage 1 and 3, Yates and McDonough and Atkinson (combined), respectively, are presented in Figure 16.

Transfer Functions for State Variables

In the theoretical development of Chapter V, the transfer functions for routing water temperature, BOD, and DO through the system sub-reach by sub-reach were designated Φ_n, ψ_n, and Ω_n, respectively. Some clarification of the form of these functions used in this investigation is in order.

Water Temperature

The actual water temperature transfer function incorporated in the computer program is, for a given flow

$$T_{n,i} = \text{TEQ} + \{[(S_{t,n} - \text{TEQ}) + \text{TRIS}_n] \cdot \text{TORD}_{n,i}\} \tag{75}$$

or

$$T_{n,i} = \text{TEQ} + (\text{FORCE}_i \cdot \text{TORD}_{n,i}) \tag{76}$$

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*M is used instead of $d_{T,n}$ for simplicity in this equation.

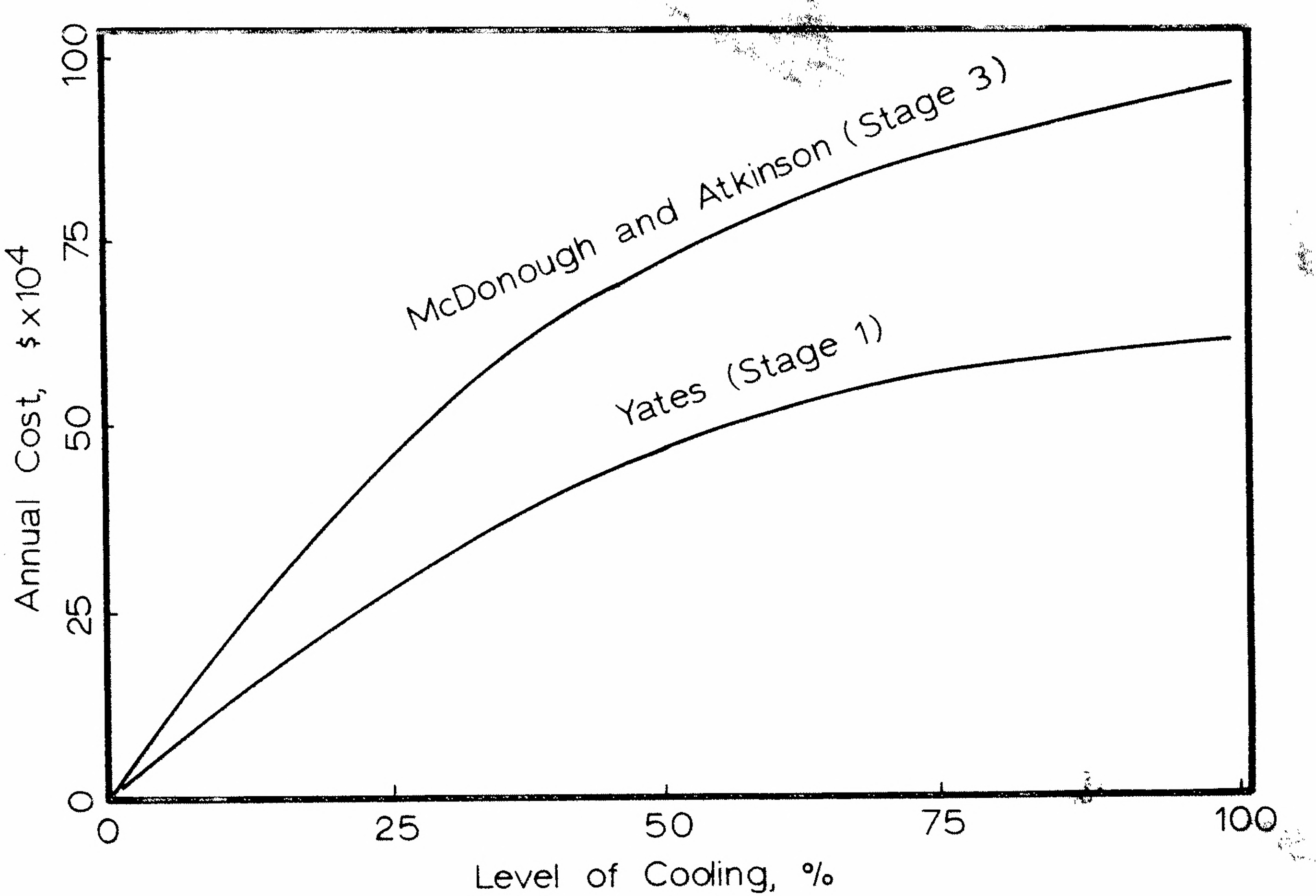

Figure 16. Annual Cooling Cost versus Abatement Level, Steam Plants

in which

$T_{n,i}$ = Water temperature at the ith point along stage n, °F.

TEQ = Equilibrium water temperature, °F.

$FORCE_i$ = Water temperature elevation above equilibrium at the top of stage n, the sum of initial elevation plus the rise due to advected heat, $\{(S_{T,n} - TEQ) + TRIS_n\}$, °F.

$S_{T,n}$ = Water temperature entering stage n from n + 1, °F.

$TORD_{n,i}$ = Unit temperature elevation ordinate for the ith point along stage n, °F/°F.

It should be noted that $T_{n,i}$ is very much a function of the flow in the stream, as the incoming water temperature ($S_{T,n}$), the $TRIS_n$, and the $TORD_{n,i}$ values are functions of flow. Therefore a different set of $TORD_{n,i}$ values must be given as input for each different flow condition. Actually, the only operation on temperature performed within the computer program proper is to scale the unit ordinates ($TORD_{n,i}$) by the elevation at the top of the stage. Calculation of the $TORD_{n,i}$ may be done external to the program because temperature is not a function of the other state variables.

The $TORD_{n,i}$ values are determined in accordance with the Velz-Gannon heat dissipation model which yields a strictly exponential decrease of excess temperature when one assumes a constant water surface width. Such an assumption may be used to simplify matters if more precise data is either unavailable or not justifiable.

Biochemical Oxygen Demand

The BOD transfer function used is an adaptation of the conventional first-order relationship, in which water temperature is considered in a dynamic manner

$$B_{n,i} = \begin{cases} (S_{B,n} + BODINCR_n)\exp(-K_{1,T} \cdot TFS_{n,i}), & \text{for } i = 1 \\ (B_{n,i-1})\exp(-K_{1,T} \cdot TFS_{n,i}), & \text{for } i = 2,3,\ldots,k_n \end{cases} \tag{77}$$

in which

$B_{n,i}$ = BOD concentration at the end of the ith sub-reach of stage n, mg/ℓ.

$B_{n,i-1}$ = BOD concentration at end of the previous (i-1)th sub-reach and entering the ith sub-reach, mg/ℓ.

$S_{B,n}$ = BOD entering stage n from n + 1, mg/ℓ.

$BODINCR_n$ = Increase in BOD concentration due to waste discharge at n, mg/ℓ.

$K_{1,T}$ = Deoxygenation rate coefficient corrected to the mean temperature in sub-reach i of stage n, 1/days.

$TFS_{n,i}$ = Time of flow through sub-reach i of stage n, days.

The use of Equation 77 results in a step-wise approximation of the BOD curve that would be produced if one allowed temperature to decay exponentially in a continuous manner. It is apparent that the precision of the approximation increases with the number of sub-reaches or steps.

Judicious selection of the number of sub-reaches must consider the trade-off between the utility of such precision and increasing computational requirements.

Dissolved Oxygen

The transfer function which relates a stage's DO response to thermal and organic waste loading is as follows

$$O_{n,i} = C_s - D_{n,i} \tag{78}$$

in which

$$D_{n,i} = \frac{K_{1,T} \cdot L_{n,i}}{K_{2,T} - K_{1,T}} [\exp(-K_{1,T} \cdot TFS_{n,i}) - \exp(-K_{2,T} \cdot TFS_{n,i})] \tag{79}$$

$$+ DEF_{n,i} \exp(-K_{2,T} \cdot TFS_{n,i})$$

in which

$$L_{n,i} = \begin{cases} S_{B,n} + BODINCR_n; & \text{for } i = 1 \\ B_{n,i-1}; & \text{for } i = 2,3,\ldots,k_n \end{cases} \tag{80}$$

and

$$DEF_{n,i} = \begin{cases} C_s - S_{0,n}; & \text{for } i = 1 \text{ and } C_s > SO_n \\ 0; & \text{for } i = 1 \text{ and } C_s \le SO_n \\ D_{n,i-1}; & \text{for } i = 2,3,\ldots,k_n \end{cases} \tag{81}$$

The nomenclature used in the above equations is defined as follows:

$O_{n,i}$ = DO concentration at end of ith sub-reach of stage n, mg/ℓ.

C_s = DO saturation concentration, a function of water temperature, mg/ℓ.

$D_{n,i}$ = DO deficit ($C_s - O_{n,i}$) at end of ith sub-reach of stage n, mg/ℓ.

$K_{1,T}, K_{2,T}$ = Deoxygenation and reaeration rate coefficients, respectively, 1/days.

$L_{n,i}$ = BOD concentration at top of ith sub-reach of stage n, mg/ℓ.

$TFS_{n,i}$ = Time of flow in the ith sub-reach of stage n, days.

$DEF_{n,i}$ = DO deficit at top of ith sub-reach of stage n, mg/ℓ.

$S_{0,n}$ = DO concentration entering stage n from the previous stage (n-1), mg/ℓ.

$D_{n,i-1}$ = DO deficit existing at end of previous sub-reach (i-1) of stage n and entering the ith sub-reach, mg/ℓ.

In order to define the DO response satisfactorily, it is necessary to have at least one sub-reach boundary in the anticipated vicinity of the sag-point.

Computer Program

For a given set of conditions, the following is input to the basic program developed in this study:

(1) Equilibrium water temperature.

(2) Increments or step sizes for state variables (temperature, BOD, and DO) and abatement facilities (cooling and treatment).*

(3) Thermal and organic waste production at each stage.

(4) Mean streamflow in each stage.

(5) Deoxygenation and reaeration rate coefficients for each stage.

(6) Water quality standards applicable to each stage (maximum temperature elevation, maximum temperature, and minimum DO concentration).**

(7) Cooling and treatment cost data for each stage.

(8) Unit temperature elevation ordinates for each stage.

(9) Time-of-flow through the sub-reaches in each stage.

(10) Upper and lower limits on state variables and abatement facilities for each stage.***

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*Symbols used will be DST, DSB, DSO, DCO, and DTR for the increments on temperature, BOD, DO, per cent cooling, and percentage treatment, respectively.

**These will be designated TRS_n, TMS_n, and DOS_n, respectively.

***The symbols used will be as follows: STL_n, SBL_n, SOL_n, CL_n, and TL_n for the lower limits on water temperature, BOD, DO, percentage cooling, and percentage treatment, respectively, and STU_n, SBU_n, SOU_n, CU_n, and TU_n for the upper limits on the same variables.

The program starts with the downstream stage ($n = 1$) and progresses upstream, stage by stage, to the Nth stage as outlined in Chapter V. Within a stage, the process is as follows.

The initial state vector entering the stage, $\bar{S}_n$, is first set equal to the lower limit on incoming water temperature (STL_n), incoming BOD (SBL_n), and incoming DO (SOL_n). Next the level of cooling is set equal to the lower limit on cooling (CL_n). With the streamflow, incoming temperature, waste heat production, and level of cooling known, one may determine the temperature rise at the top of the reach and resulting mixed temperature. These are compared to the temperature rise and maximum temperature standards (TRS_n and TMS_n) applicable to the reach. If a temperature standard is violated, the level of cooling in increased by the cooling level increment (DCO) and the calculations and tests made again. When the temperature standards are met, the temperature profile is obtained by scaling the unit ordinates by the elevation at the top of the reach. Then the temperature-dependent parameters are calculated for each sub-reach. These are the mean temperature (MT_i), the deoxygenation and reaeration rate coefficients ($K_{1,i}$ and $K_{2,i}$), and the saturation DO concentration (SAT_i).

Now the level of treatment is set at its lower level (TL_n), and the DO deficit and mixed BOD concentration at the top of the stage are determined. With all the parameters of the DO response function known for the initial sub-reach, the process steps down the stage, sub-reach by sub-reach, checking the DO concentration at each sub-reach boundary against the DO standard for the stage. If at any point the DO is less than DOS_n, the level of treatment is incremented by the treatment increment (DTR); and the resulting DO response is determined and checked.

When a combination of abatement measures ($\bar{D}_n$) yields a feasible solution (i.e., no violations), the resulting output vector ($\bar{\bar{S}}$) is stored in a matrix corresponding to the particular $\bar{S}_n$ and $\bar{D}_n$. The $\bar{\bar{S}}$ allows one to enter the matrix corresponding to the (n-1)th stage and determine the minimum cost for such an input to the (n-1)th stage. This optimal (n-1)-stage cost, plus the cost associated with the $\bar{D}_n$ is also placed in temporary storage. The total n-stage cost is determined in a similar manner for all feasible $\bar{D}_n$* as the program proceeds to step on cooling and treatment on up to their upper limits. When all $\bar{D}_n$ associated with an $\bar{S}_n$ have been investigated, the $\bar{D}_n$ that has resulted in the minimum total n-stage cost is placed in permanent storage corresponding to the $\bar{S}_n$, as well as the optimal n-stage cost and the resulting $\bar{\bar{S}}$.

New $\bar{S}_n$'s are obtained by incrementing $S_{O,n}$ by the incoming DO increment. The entire process is repeated for each of these new input vectors. When $S_{O,n}$ reaches its upper limit, incrementing of $S_{B,n}$ by the incoming BOD increment commences. When the upper limit on the incoming BOD is reached, $S_{T,n}$ is incremented by the incoming temperature increment.

In this manner, all possible abatement programs at stage n ($\bar{D}_n$) are investigated for every initial state vector ($\bar{S}_n$). When the

*$\bar{D}_n$ that result in a violation of a water quality standard are a assigned an arbitrarily large cost to prevent their further consideration.

process for a stage is completed, one has an optimal n-stage abatement policy for each possible input vector. An abbreviated flow diagram flow diagram is shown in Figure 17. For details on the procedure, one is referred to Appendix A where a listing of the basic ALGOL program for use on a Univac 1108 computer is presented.

Example Computer Runs for Chattahoochee River Basin

To illustrate the use of the methodology developed in the course of this investigation, the results of several computer runs will be presented based on three-day, ten-year September Chattahoochee River low-flow and once-in-ten-years adverse September meteorology for the basin. The initial run was made using rather coarse increments on the state variables;* successive runs were made with finer increments on inputs and smaller ranges on inputs and abatement measures, determined from previous runs. Increments of 2 per cent were considered adequate for cooling and treatment for all runs.

Cost data for cooling at stages 1 and 3 and treatment at stage 4 were abstracted from the cost versus abatement level curves presented in Figures 15 and 16. Mean flow in the stages was taken from Figure 6. The initial conditions ($\bar{S}_4$) were 80.7°F, 2.0 mg/ℓ, and 6.7 mg/ℓ for initial water temperature, BOD, and DO, respectively. A summary of input data is presented in Tables 9 and 10.

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*0.5°F, 2 mg/ℓ, and 0.5 mg/ℓ for DST, DSB, and DSO, respectively.

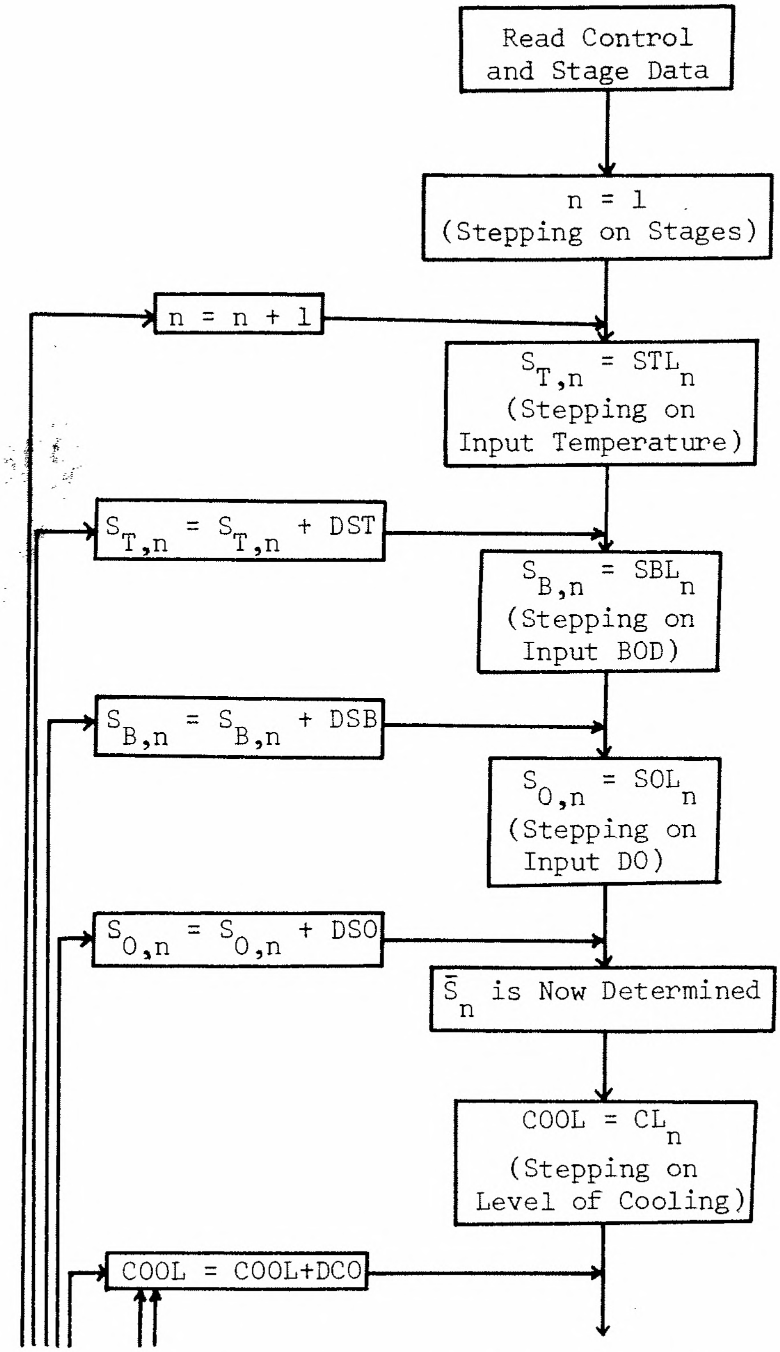


Figure 17. Abbreviated Flow Diagram of Program Which Determines the Optimal Abatement Policy for a River Basin Modeled as an N-Stage Process

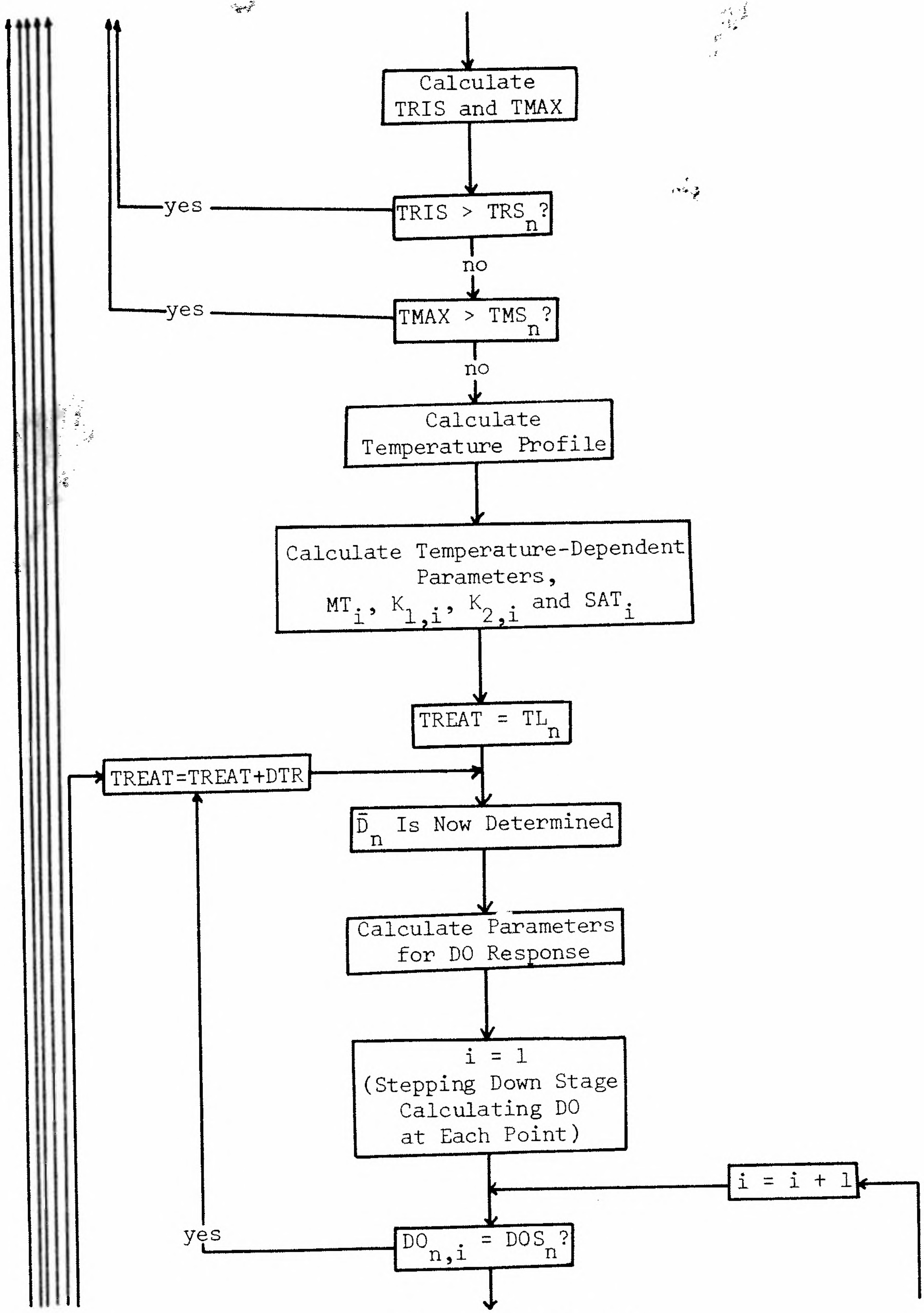


Figure 17 (Continued).

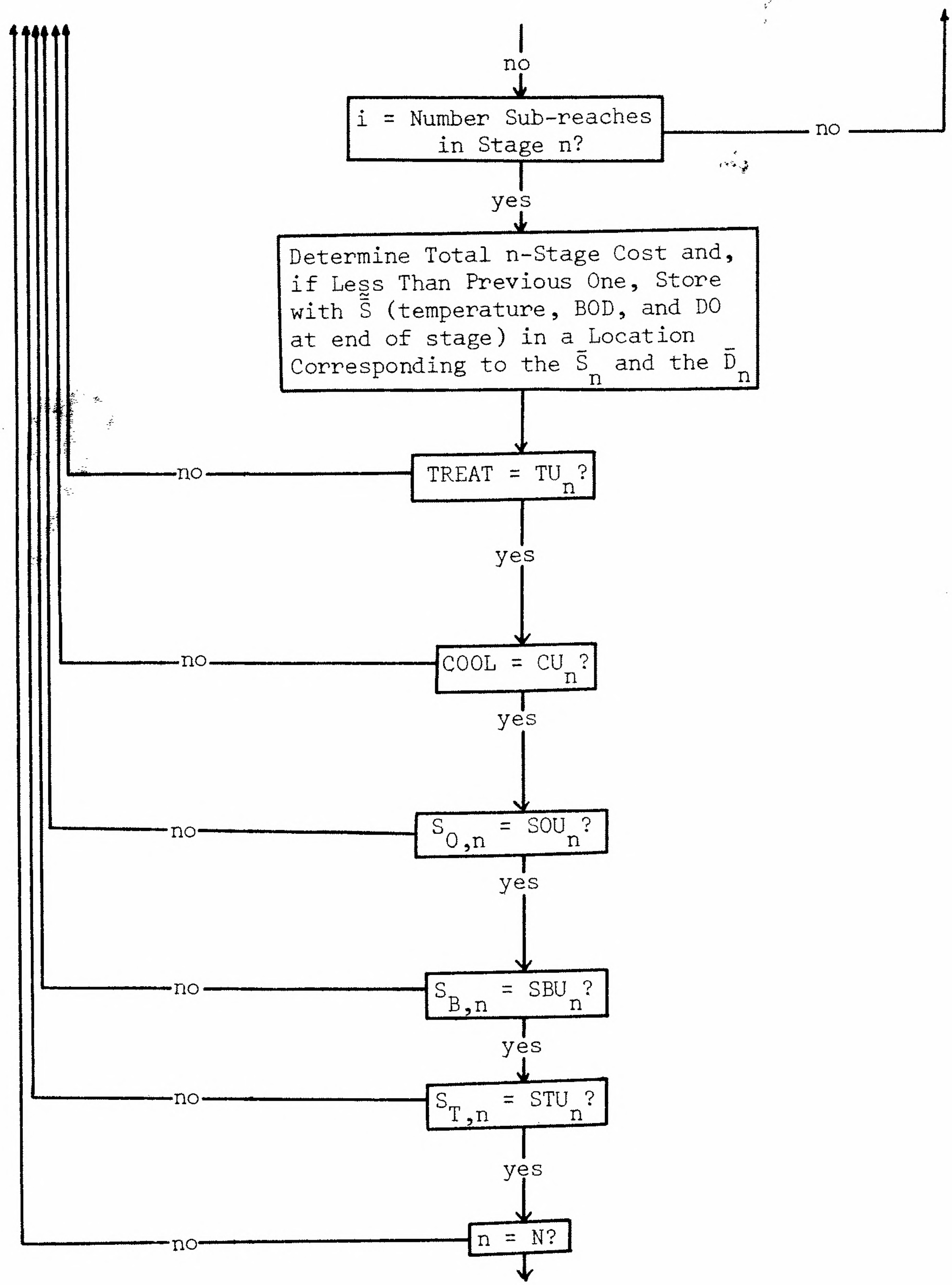


Figure 17 (Continued).

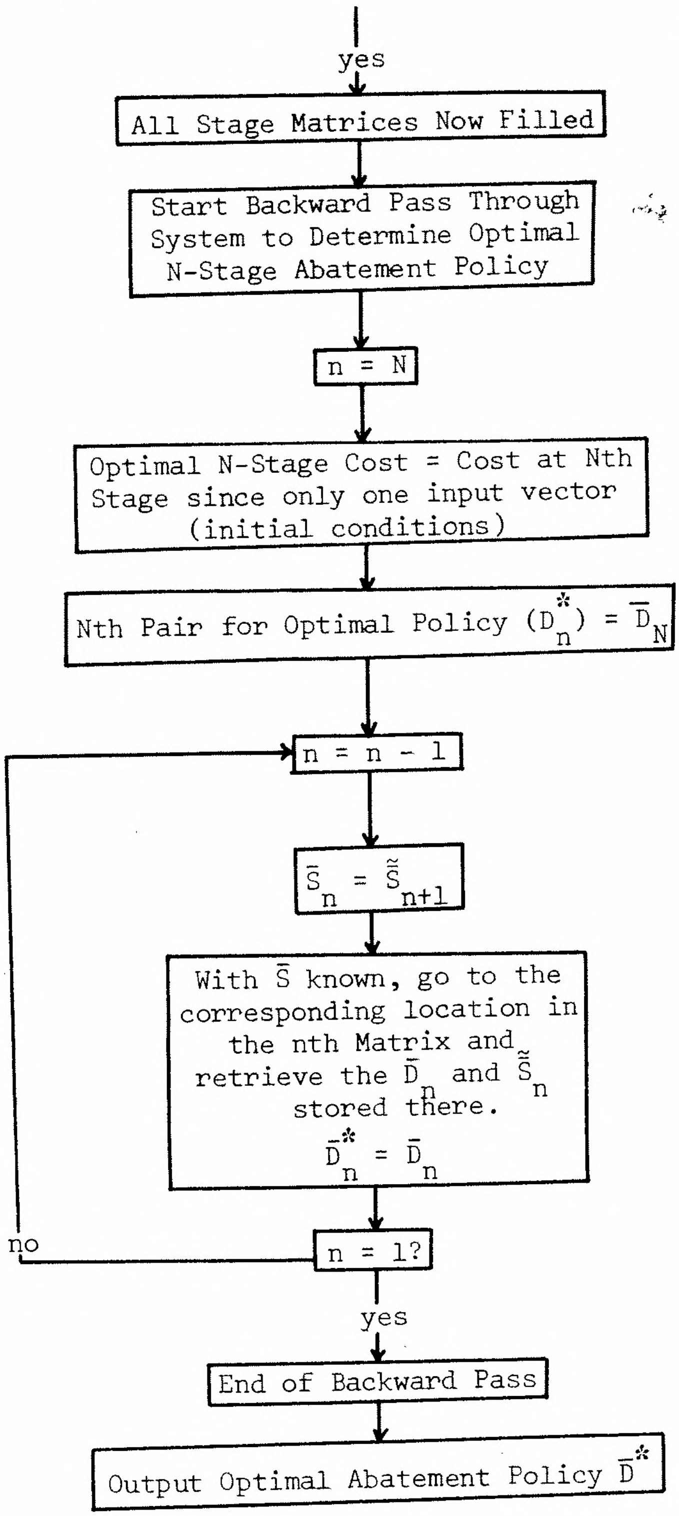


Figure 17 (Concluded).

Table 9. Stage Input Data for One-in-Ten September Chattahoochee River Computer Runs

Stage, n	Heat, T_n 10^9Btu/hr	BOD, B_n 10^3#/day	Flow, Q_n (cfs)	$K_{1,20}$ 1/days	$K_{2,20}$ 1/days	Number of Sub-Reaches, NSR_n	Dis. (mi)	Standards			Input and Abatement Ranges									
								TRS_n (°F)	TMS_n (°F)	DOS_n mg/ℓ	STL_n (°F)	STU_n (°F)	SBL_n mg/ℓ	SBU_n mg/ℓ	SOL_n mg/ℓ	SOU_n mg/ℓ	TL_n (%)	TU_n (%)	CL_n (%)	CU_n (%)
4	0	110	820	1.0	0.9	1	1	10	93	3.0	80.7	80.7	2.0	2.0	6.7	6.7	0	0	0	100
3	2.4	0	870	1.0	0.9	8	41	10	93	3.0	80.7	80.7	1.0	21.0	3.0	8.0	0	100	0	0
2	0	0	885	1.0	0.9	2	3	10	93	4.0	80.7	82.2	0.0	2.0	4.0	8.0	0	0	0	0
1	1.6	0	920	1.0	0.9	9	44	10	93	4.0	80.7	81.7	0.0	2.0	4.0	8.0	0	100	0	0

Table 10. Unit Temperature Ordinates and Time-of-Flow in Sub-Reaches for One-in-Ten September Chattahoochee River Computer Runs

	Sub-Reach, i									
	1	2	3	4	5	6	7	8	9	10
$TORD^*_{4,i}$	1.00	0.94								
$TFS^*_{4,i}$	0.05									
$TORD_{3,i}$	1.00	0.89	0.79	0.68	0.53	0.42	0.32	0.19	0.09	
$TFS_{3,i}$	0.10	0.10	0.10	0.20	0.20	0.20	0.40	0.57		
$TORD_{2,i}$	1.00	0.93	0.83							
$TFS_{2,i}$	0.07	0.10								
$TORD_{1,i}$	1.00	0.87	0.76	0.67	0.52	0.40	0.31	0.16	0.08	0.04
$TFS_{1,i}$	0.10	0.10	0.10	0.20	0.20	0.20	0.50	0.50	0.50	

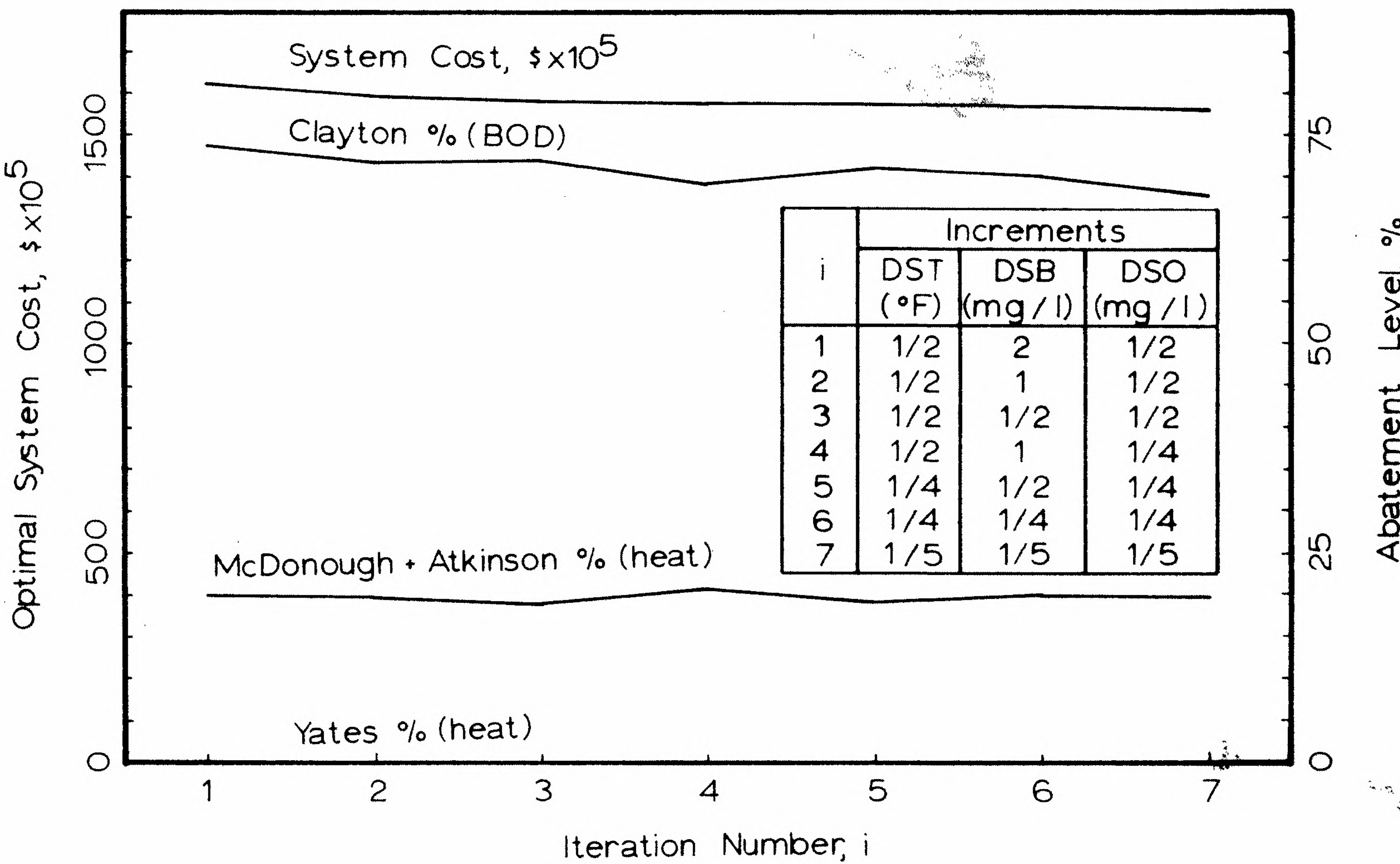
*Units: $TORD_{n,i}$, °F/°F $TFS_{n,i}$, days.

Optimal abatement policies for each run along with the resulting minimum basin costs are presented in Figure 18. General decrease in cost with successive runs indicates that the true optimum is being approached. Also, the decreasing amount of improvement in the objective function (total cost) should be noted, indicating the flatness of the region around the point of optimality.

The DO profiles for the system resulting from the policies determined in the various runs are shown in Figure 19. It should be noted that excess assimilative capacity may exist or violations may occur in profiles for the first runs with coarse increments. This is due to the round-off of the $\tilde{\bar{S}}_{n+1}$ to the $\bar{S}_n$ in the backward pass of the program. As precision or resolution increases, this phenomenon tends to decrease.

It is of interest to note that no significant interaction between thermal and organic wastes occurs for the conditions of these runs. The level of cooling prescribed is simply that required to prevent violation of the temperature standards. This is due to the fact that, though the initial rise in stream temperature due to a thermal waste influx may be 10°F, significant dissipation of the heat occurs by the time the DO sag-point is reached. The elevated temperature does, however, tend to move the sag-point upstream because of the effect on the rate coefficients. The effect of the heat on the DO curve is noticeable at the Plant Yates outfall where a dip occurs in the recovery zone.

It should be pointed out that the heat sources in the Chattahoochee River system studied presently have no cooling facilities.



i	Increments DST (°F)	DSB (mg/l)	DSO (mg/l)
1	1/2	2	1/2
2	1/2	1	1/2
3	1/2	1/2	1/2
4	1/2	1	1/4
5	1/4	1/2	1/4
6	1/4	1/4	1/4
7	1/5	1/5	1/5

Figure 18. Optimal Cost and Abatement Policy as a Function of Iteration Number

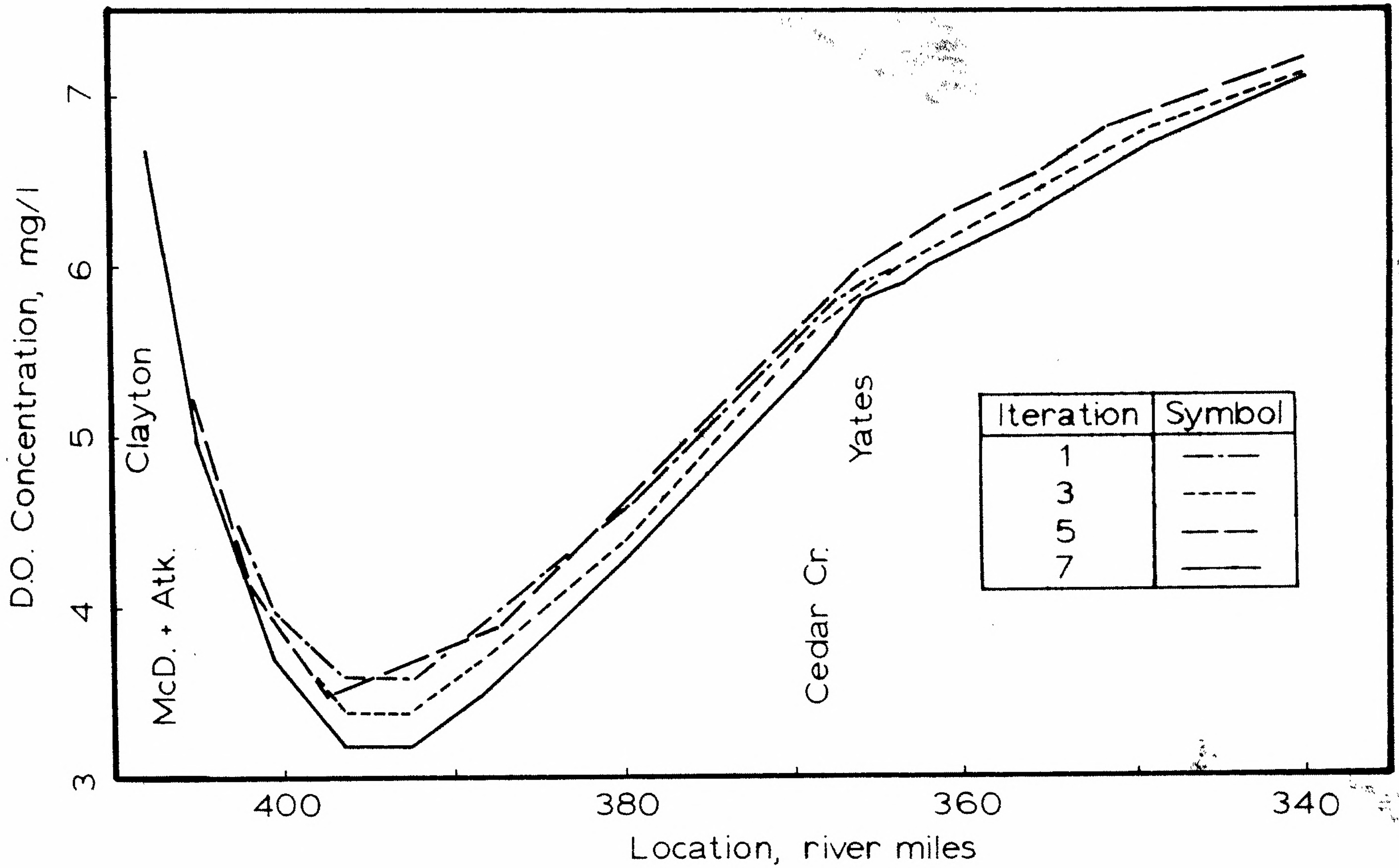


Figure 19. DO Profiles

Since Plants McDonough and Atkinson (combined) can raise the temperature of the Chattahoochee River 12.3°F for the one-in-ten September conditions, one may conclude that the temperature standards would be violated under such conditions.

CHAPTER VII

ANALYSIS OF SYSTEM SENSITIVITY TO PARAMETER VARIATION

Although significant benefits toward improved, more efficient water quality planning and management may be derived from the application of the methodology developed herein, extensions to the basic program provide greatly increased guidance to policy makers.

To this point, system parameters have been considered fixed; and this has resulted in the investigation of only one set of system conditions. In this chapter, several of the principal system parameters will be varied systematically; and the cost and abatement policy response will be investigated.

Variation of Streamflow

The great importance of streamflow to water quality is well known. Increased flow provides greater dilution for thermal as well as organic pollutants, but some possibly adverse effects may result. The DO sag-point will move downstream with increasing flow; also, though the initial temperature rise will be less, the zone of temperature elevation will become longer. These phenomena are attributable to decreasing time of travel that results from increasing flow velocities.

Since flow augmentation for quality control has become a legitimate purpose in water resources development, increased consideration has been given to flow augmentation as an alternative to more local abatement. Quite obviously, one would like to determine the basin's abatement costs as a function of flow rate so that an optimal, or at least reasonable, balance between the two may be found. The sum of cost allocated to flow augmentation and local abatement costs should be minimized.

In some cases, conflicts develop between power or flood control interests and downstream municipalities or industries. The operating policies of the former may not serve the best interests of the latter with regard to water quality considerations. Such has been the case in the Chattahoochee River basin with the conflict between the City of Atlanta and the Corps of Engineers.

Experiments were conducted using the model developed with flows both above and below the three-day, ten-year September low-flow in increments of 100 cfs. All other system parameters except K_2 were held constant. The increases and decreases are uniform for the entire system,* reflecting changes in the operating policy of Buford Dam above Atlanta. An additional analysis was made for three-day, ten-year July low-flow hydrology and one-in-ten-year adverse July meteorology.

Water quality conditions are usually at their worst sometime in the period from July through October. This corresponds to the months of low flow and warm weather. Adverse flow conditions usually occur during late summer or early fall, whereas, adverse meteorology is

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*That is, the entire system hydrograph is shifted up or down.

encountered in mid-summer. The month of September was selected to illustrate the use of the methodology developed in this study for low-flow conditions; the month of July to illustrate its use for warm weather conditions. In an investigation that considers temperature as a prime quality parameter, as well as its effects on other system parameters, both adverse hydrology and meteorology must be considered.

For the September experiments, flow was varied from 300 cfs below the three-day, ten-year September figures to 400 cfs above in increments of 100 cfs. The resulting optimal system costs and abatement policies* are shown as functions of flow in Figure 20. The response of cost to changing flow is rather significant, a 35 per cent increase in system cost for a 300 cfs decrease in flow and a 24 per cent decrease in cost for a 400 cfs increase in flow. As one would expect, the improvement rate in system cost decreases with increasing flow, indicating that the marginal utility of successive flow increments is decreasing.

The effect of a variable flow on the prescribed level of abatement at Clayton is rather small, percentagewise and costwise, the 300 cfs decrease raising treatment 10 per cent and cost by 9 per cent and the 400 cfs increase lowering treatment by 10 per cent and cost by only 3 per cent. The response of the steam-electric plants is linear and more significant. The level of cooling prescribed is that necessary to

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*The responses referred to are responses in optimal costs and optimal policies. Also, comparisons of responses in subsequent sections are comparisons of optimal costs and policies for different conditions.

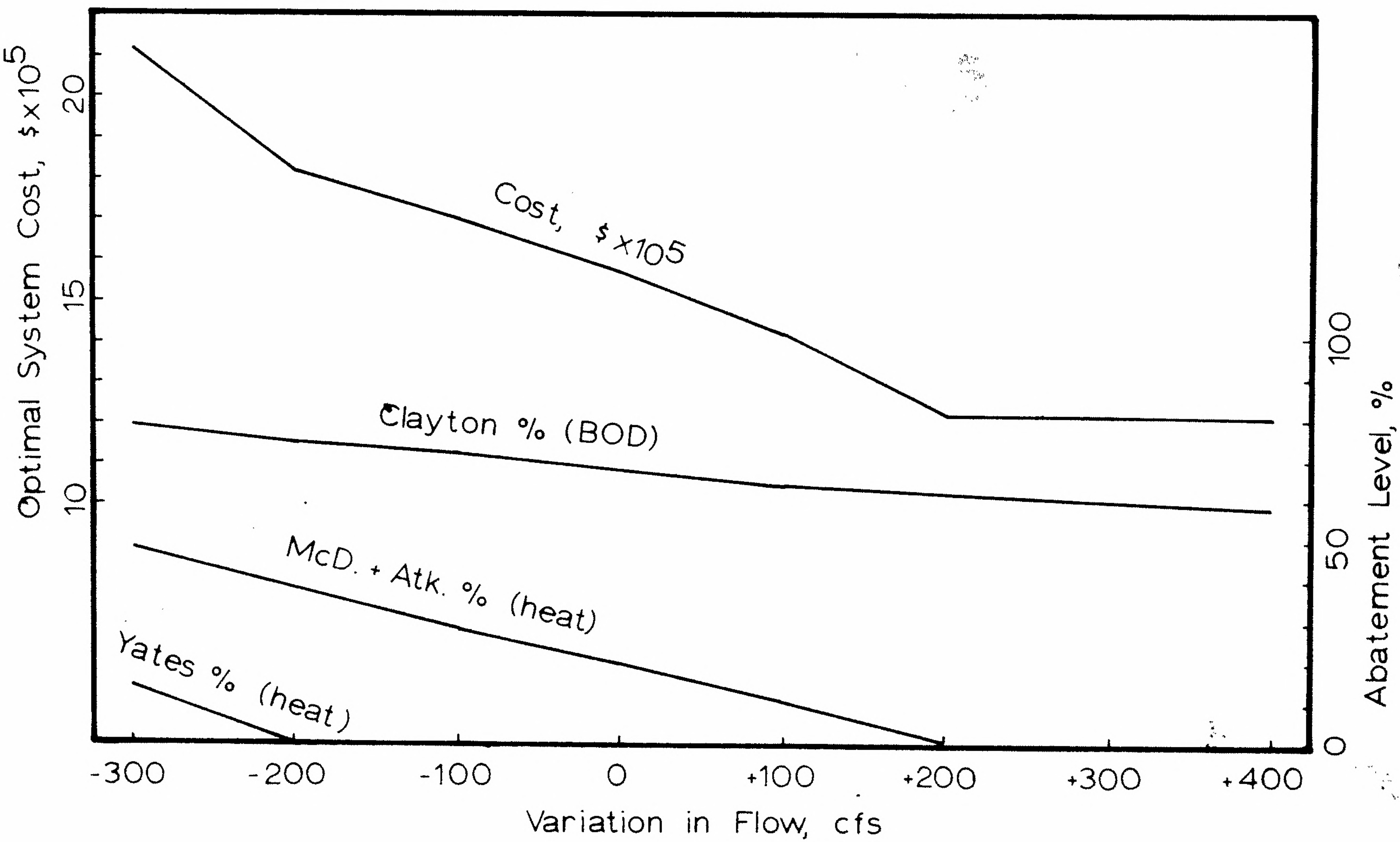

Figure 20. System Response to Flow Variation (September)

meet the temperature standards.* This indicates that, for a wide variation of flows, the existing Chattahoochee River system should experience no interaction between thermal and organic wastes, provided that the levels of cooling and treatment indicated in Figure 20 are employed.

The DO response curves for the various flows are shown in Figure 21. Presently, there are no cooling facilities; and the City of Atlanta provides only primary treatment.

For the July runs, flow was varied from 400 cfs below the three-day, ten-year July figures to 300 cfs above, again in increments of 100 cfs. The resulting optimal system costs and abatement policies are shown as functions of flow in Figure 22. The effect on total system cost is linear and considerably less pronounced than for the September runs, an increase of 12 per cent in system cost for a 400 cfs flow decrease and a 5 per cent decrease in cost for the 300 cfs increase in flow. The general level of the cost curve is higher than for September, indicating that, if temperature standards are to be met and enforced, July conditions must be investigated.

The effect of a varying flow on Clayton abatement and the general level is essentially the same as for September conditions, whereas, the level of cooling required is significantly higher. The increased level of total system cost is attributable to the higher levels of cooling necessitated by the higher one-in-ten July equilibrium water temperature of 88.5°F. Again, there appears to be no interaction

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*For September condition, the temperature rise standard, TRS_n, is the controlling temperature constraint.

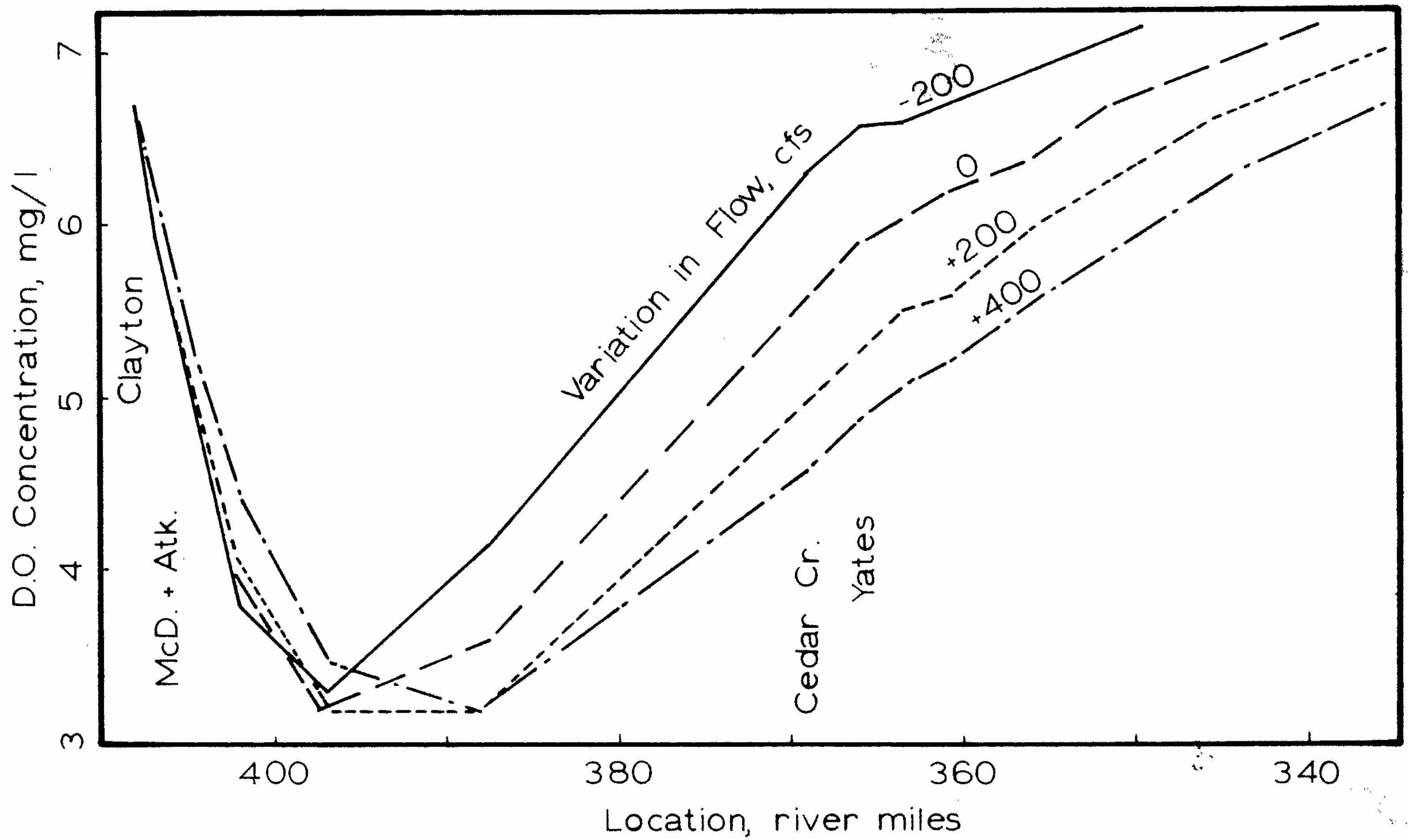

Figure 21. DO Profiles for Flow Variation (September)

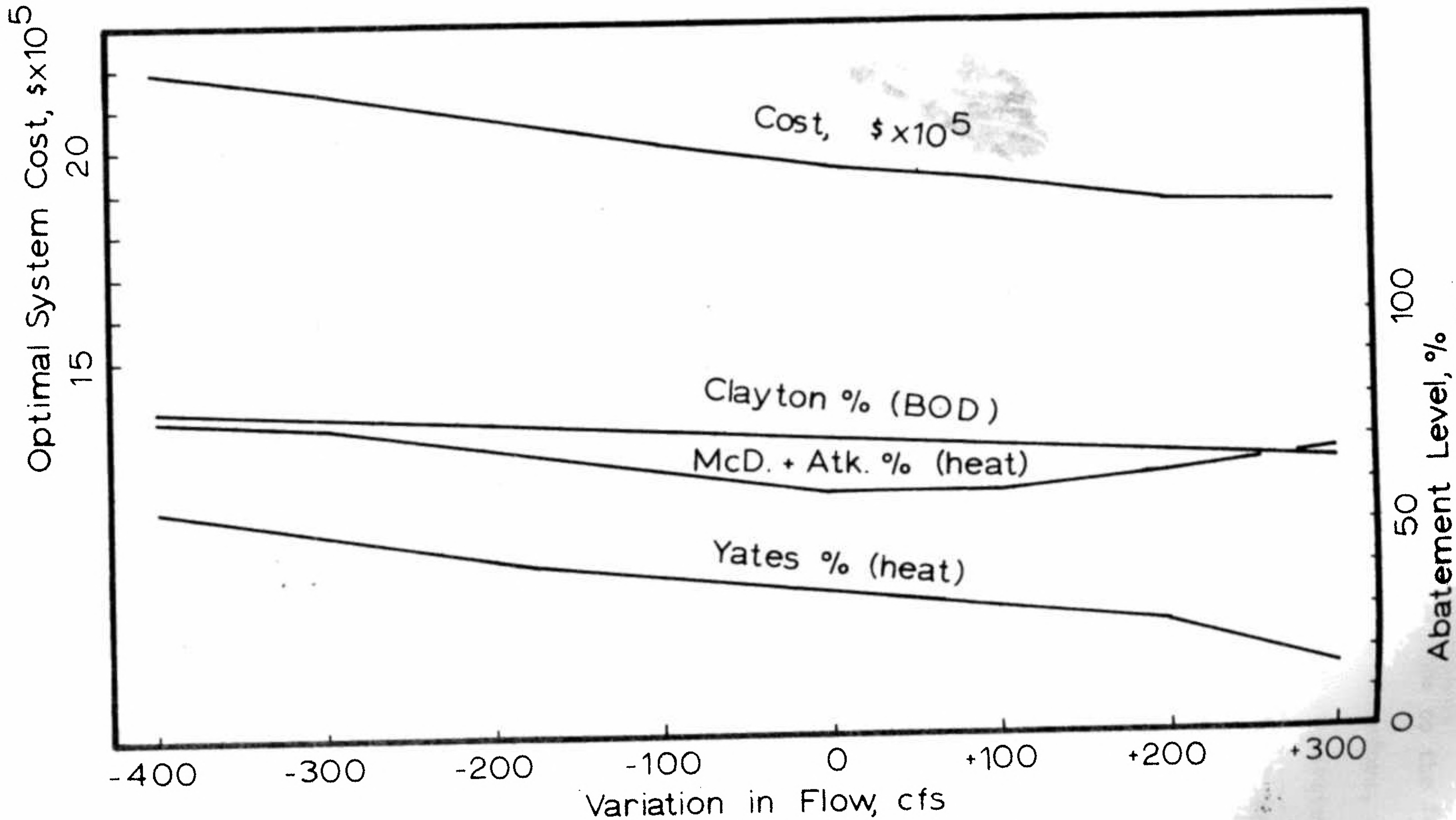

Figure 22. System Response to Flow Variation (July)

between thermal and organic wastes. This is due to the fact that the maximum allowable temperature (TMS_n) is 93°F, therefore, although the temperature rise standard (TRS_n) is 10°F, water temperature can be raised only 4.5°F. By the time the critical DO region is reached, dissipation of heat has reduced temperature elevation to a rather insignificant level. The DO sag will be somewhat more upstream than without the heat from McDonough-Atkinson, and the process of satisfying the BOD will be accelerated. The DO response curves for the various flows are shown in Figure 23.

Variation of the Deoxygenation Coefficient

The accuracy of the determination of the deoxygenation coefficient, K_1, is often less than what might be desired. Since K_1 is so important in water quality matters it is of interest to investigate the system cost and abatement policy response to systematic variation of K_1.

For September conditions, the K_1 value used in this investigation, 1.0/day, was scaled by factors ranging from 0.25 to 2.00 in 0.25 increments. This provides a rather wide range of variation. All other parameters were held constant. The resulting optimal system costs and abatement policies are shown in Figure 24 as a function of K_1.

The effect of variation of K_1 on optimal system cost is slight. The decrease in cost for a 75 per cent decrease (below the base of 1.00/day) of K_1 is 11 per cent, the increase in cost for a 100 per cent increase of K_1 is 8 per cent. This is indicative of relative insensitivity of optimal cost to changes in K_1. If one were within ± 50 per

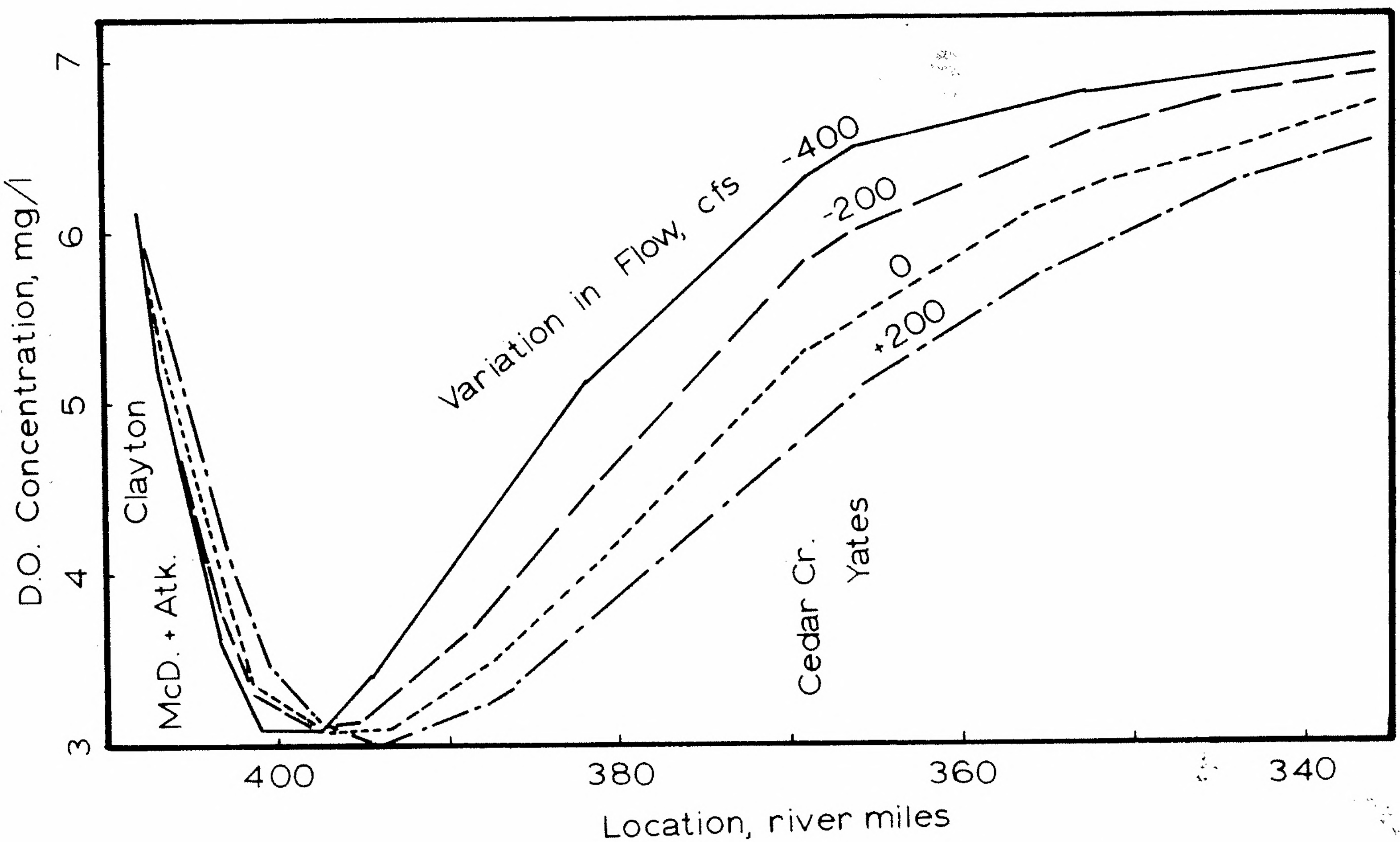

Figure 23. DO Profiles for Flow Variation (July)

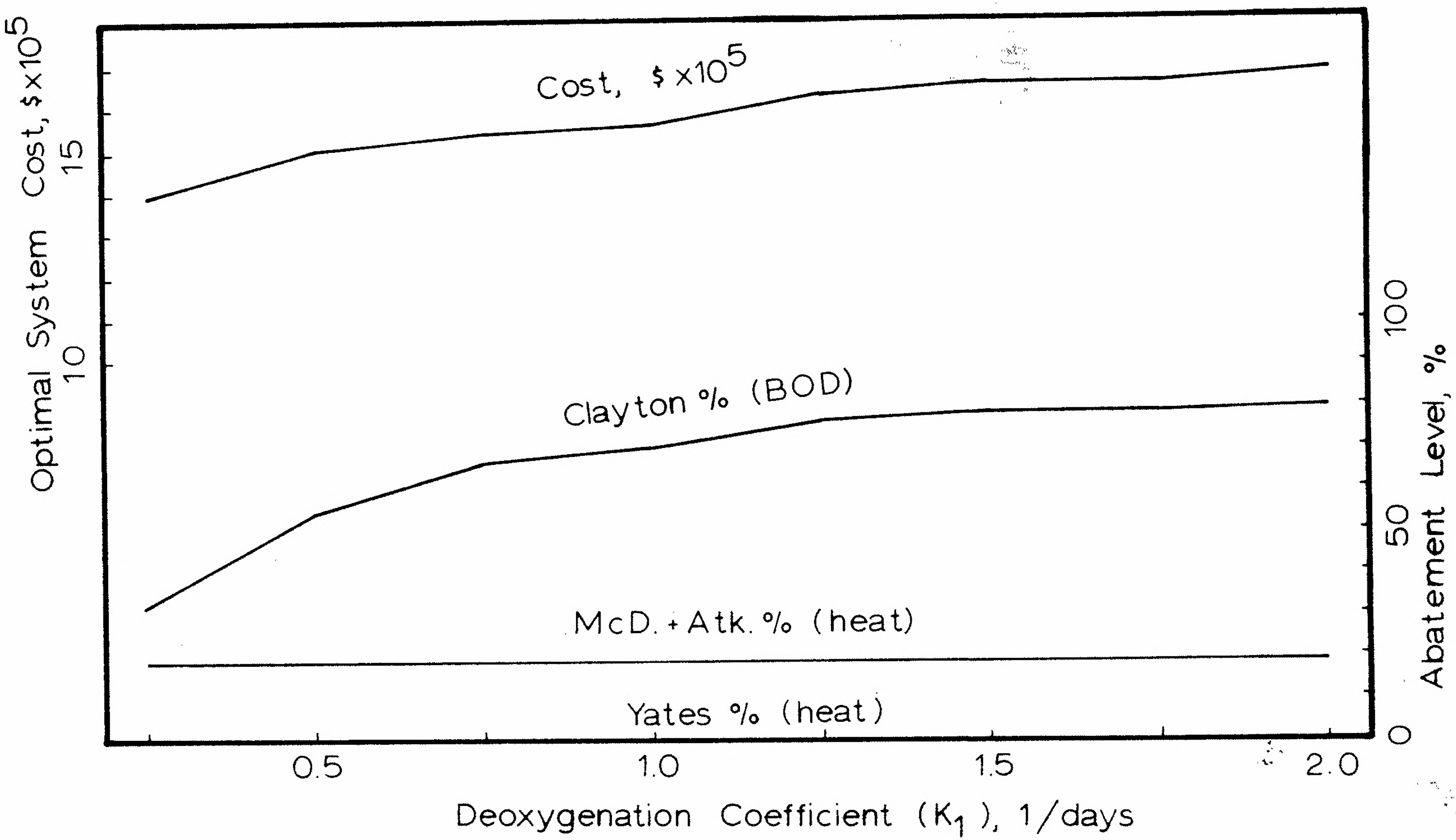

Figure 24. System Response to K_1 Variation (September)

cent of the true value, the error in optimum total system cost would be less than 10 per cent for the system as modeled in this investigation.

Changes in K_1 have no effect on required cooling but a significant effect on treatment at Clayton. Treatment increases at a decreasing rate with increasing values of K_1. The DO response profiles for the various K_1 conditions are shown in Figure 25.

For July conditions, results were very similar. The system cost curve was at a higher level than for September due to increased cooling costs but was still only slightly responsive to changing K_1 values. The treatment level at Clayton remained at the September level, and the levels of cooling were constant at the level required to meet temperature standards. This indicates a lack of interaction among the system parameters over a wide range of K_1 values. Results of the July runs are shown in Figures 26 and 27.

Variation of the Reaeration Coefficient

Inability to accurately determine the value of the reaeration coefficient, K_2, and variation of values obtained through the use of existing models for K_2 are often cited as reasons for questioning or rejecting the results of a water quality investigation. K_2 is generally thought to be the most sensitive parameter in such an investigation. In this study, K_2 was varied both above and below the base value of 0.90/day by scaling with factors of from 0.25 to 2.00 in steps of 0.25 to determine, for the Chattahoochee River basin, the

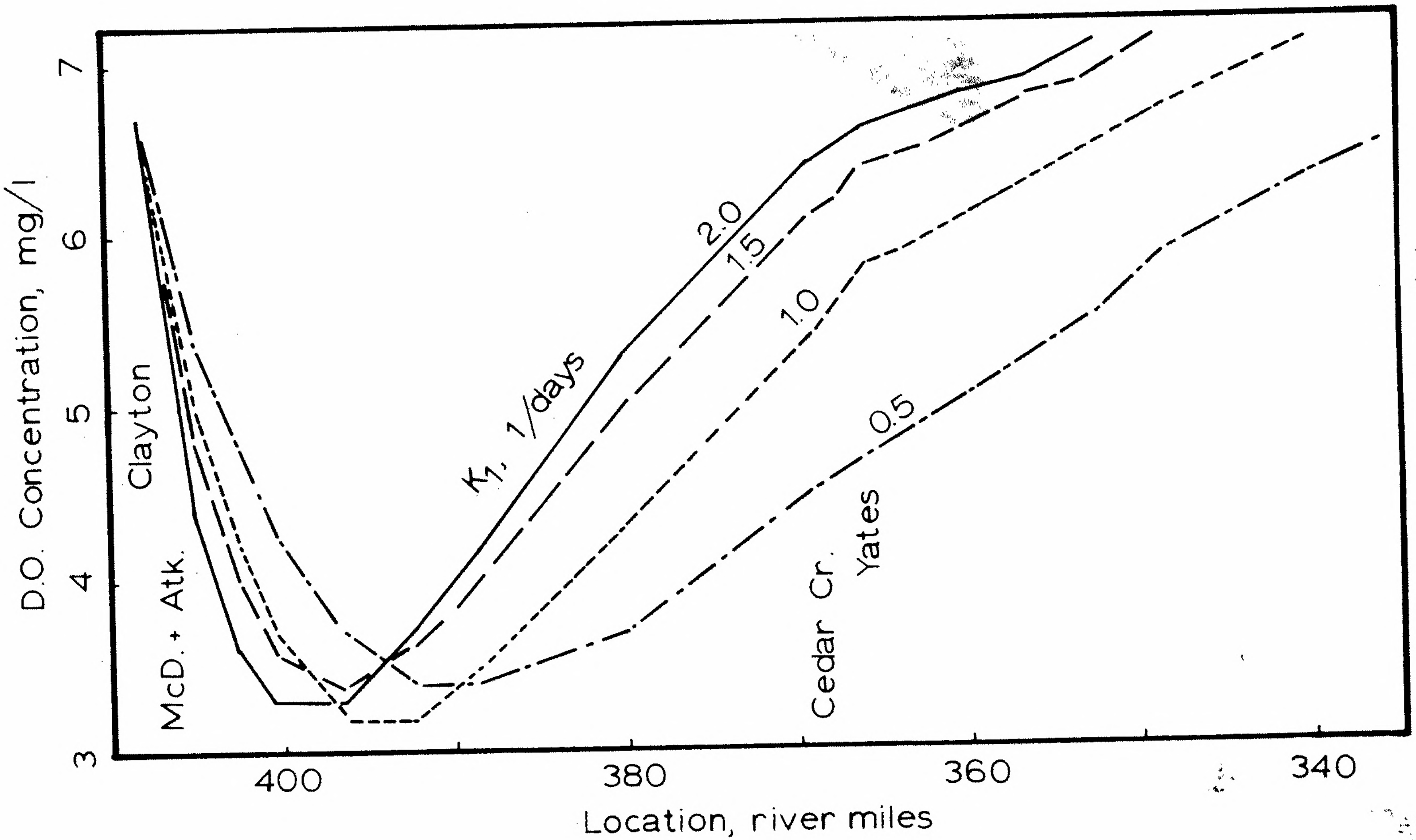

Figure 25. DO Profiles for K_1 Variation (September)

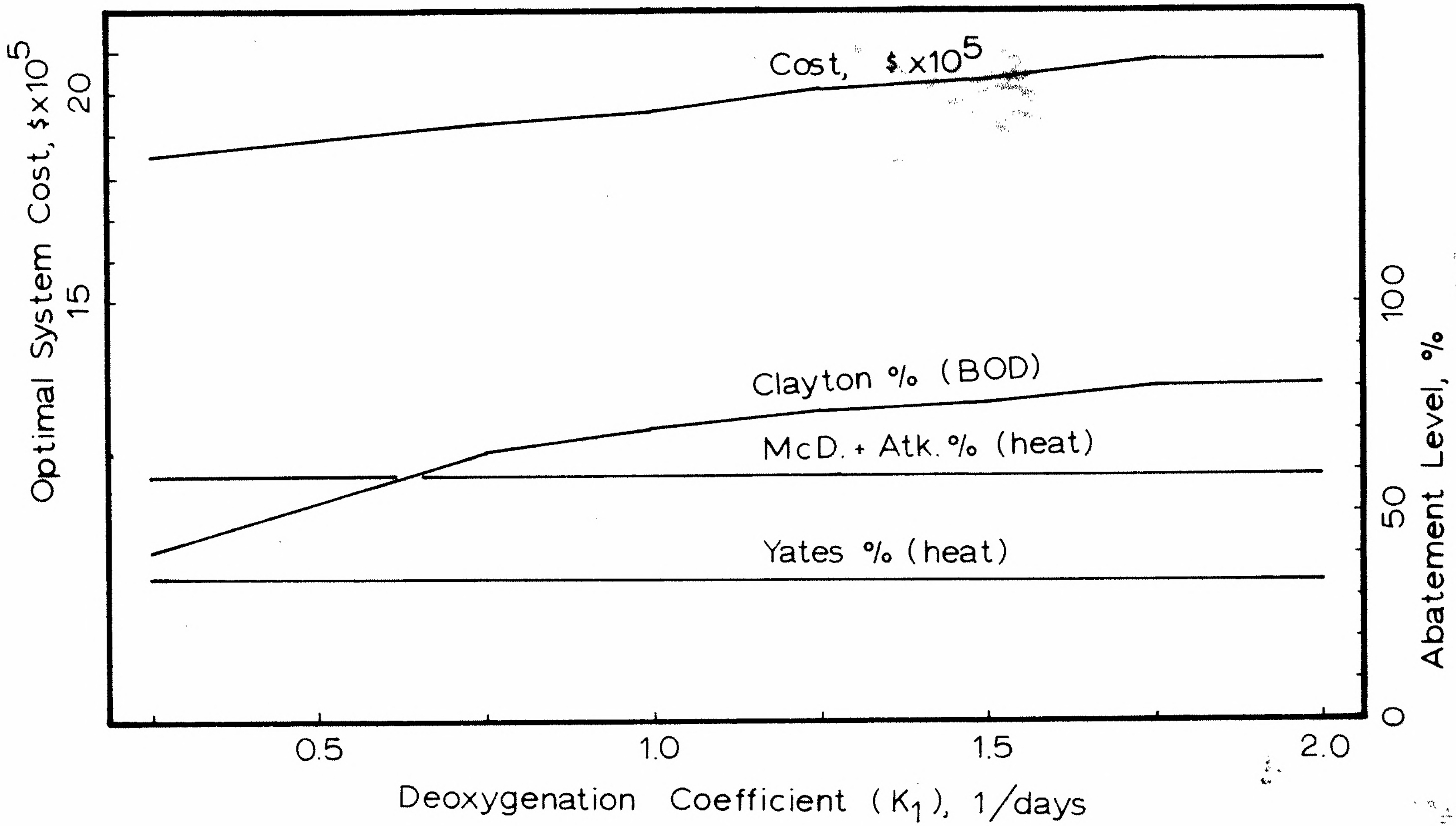

Figure 26. System Response to K_1 Variation (July)

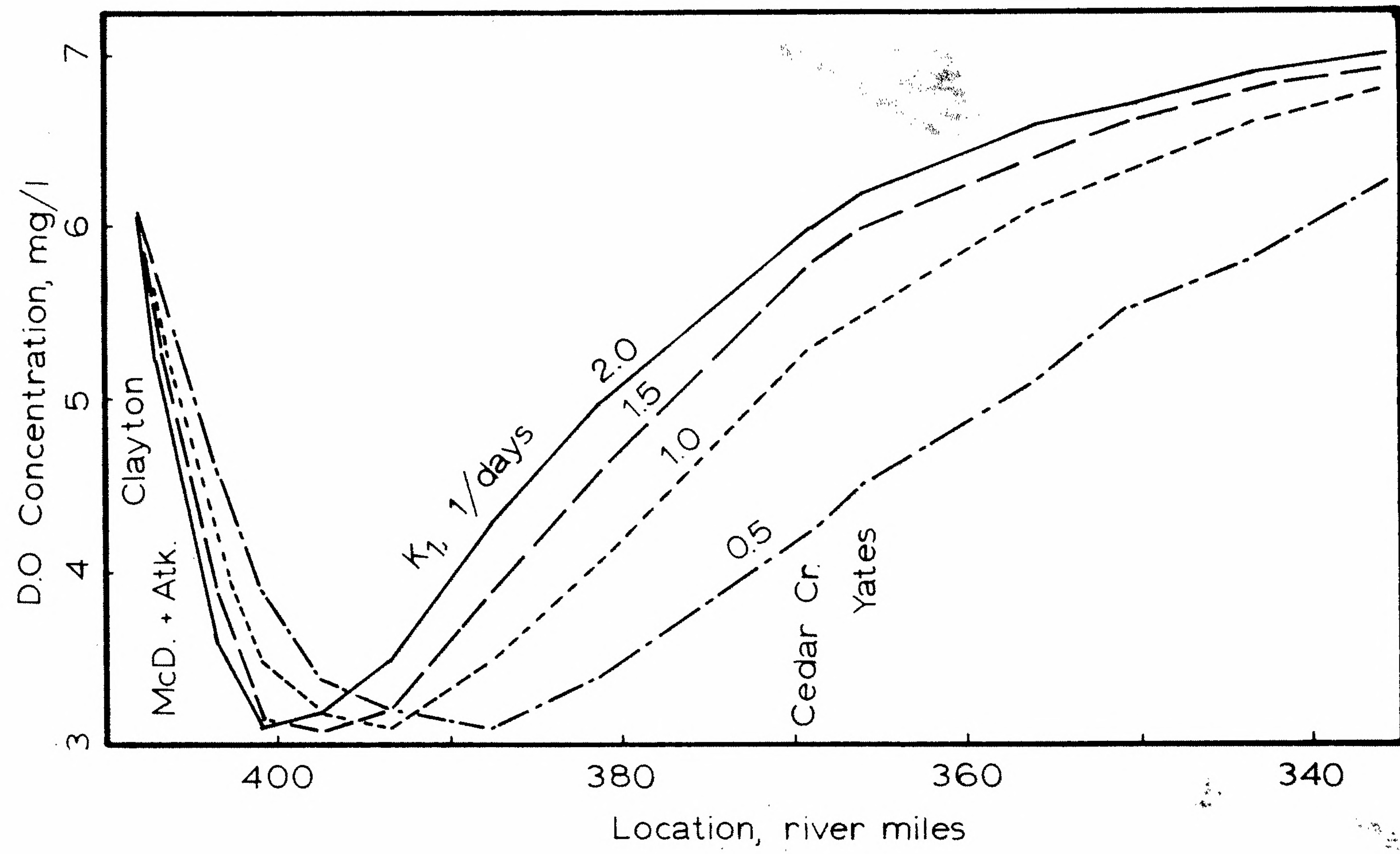

Figure 27. DO Profiles for K_1 Variation (July)

response of optimal system cost and abatement policies to different values of K_2. All other parameters were held constant.

For September conditions, the results are presented in Figures 28 and 29. System cost decreases at a decreasing rate with increasing values of K_2. The increase in optimal total system cost for a 75 per cent decrease in K_2 is 28 per cent; the decrease in cost for a 100 per cent increase in K_2 is 5 per cent.

The response of the required treatment at Clayton is practically linear, varying from 92 per cent at a 75 per cent reduction of K_2 to 56 per cent for a 100 per cent increase in K_2. The required levels of cooling are constant over the range of K_2 investigated, again indicating a lack of interaction between thermal and organic wastes. The degree of sensitivity appears to be approximately the same for K_1 and K_2 for September conditions.

The results for July conditions are presented in Figures 30 and 31. The response of system cost to variation of K_2 appears to be great at low values of K_2; otherwise, the response is similar to that for September conditions but shifted upward, reflecting the increased level of required cooling. The response and level of treatment at Clayton is practically identical with the September curve; the 2 to 3 per cent increase in treatment for July at low K_2 values accounts for the large difference in cost. When the Clayton treatment level exceeds 95 per cent, the cost increases greatly.

This situation tends to reinforce the contention of no interaction in the existing Chattahoochee River System under study when all

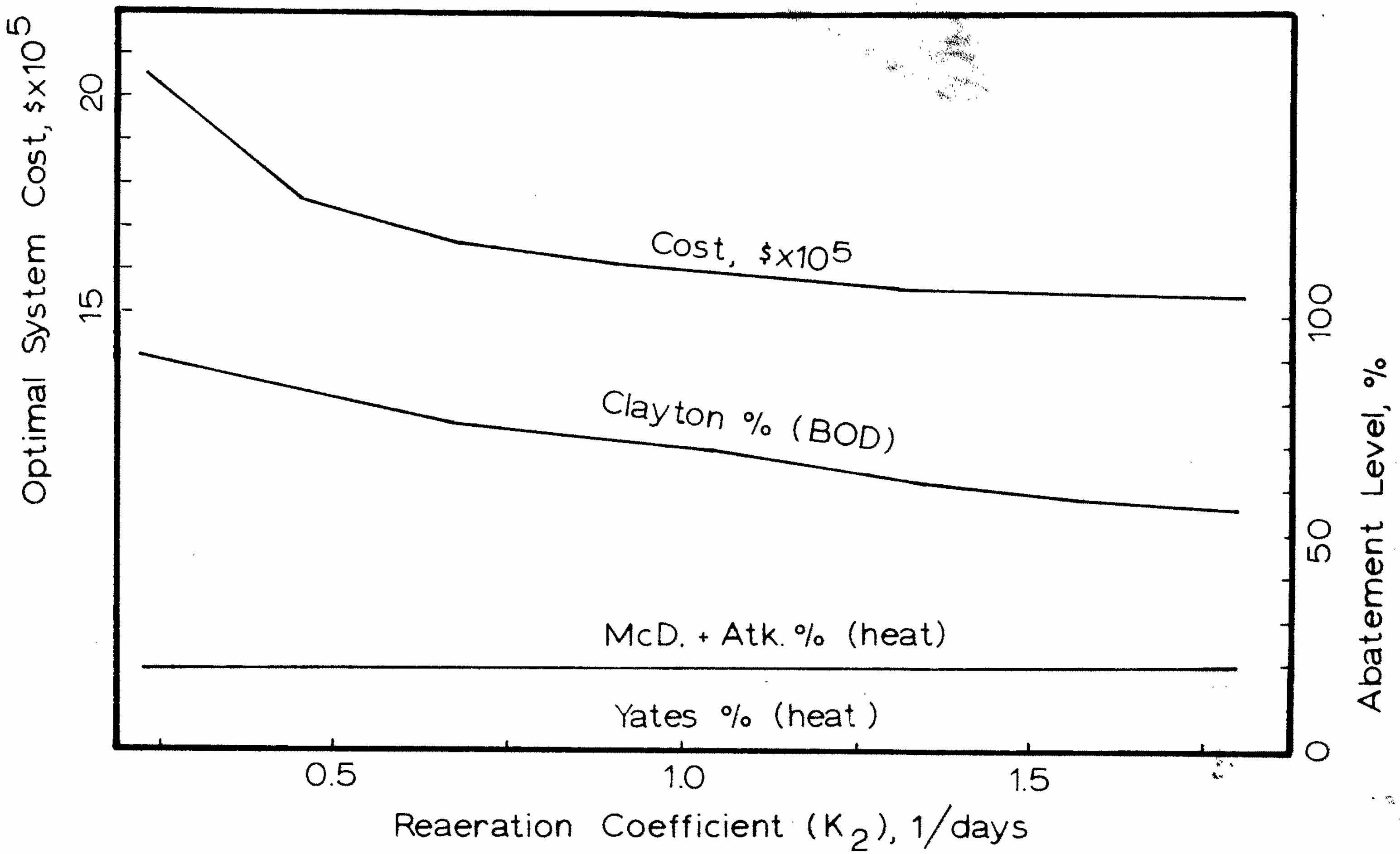

Figure 28. System Response to K_2 Variation (September)

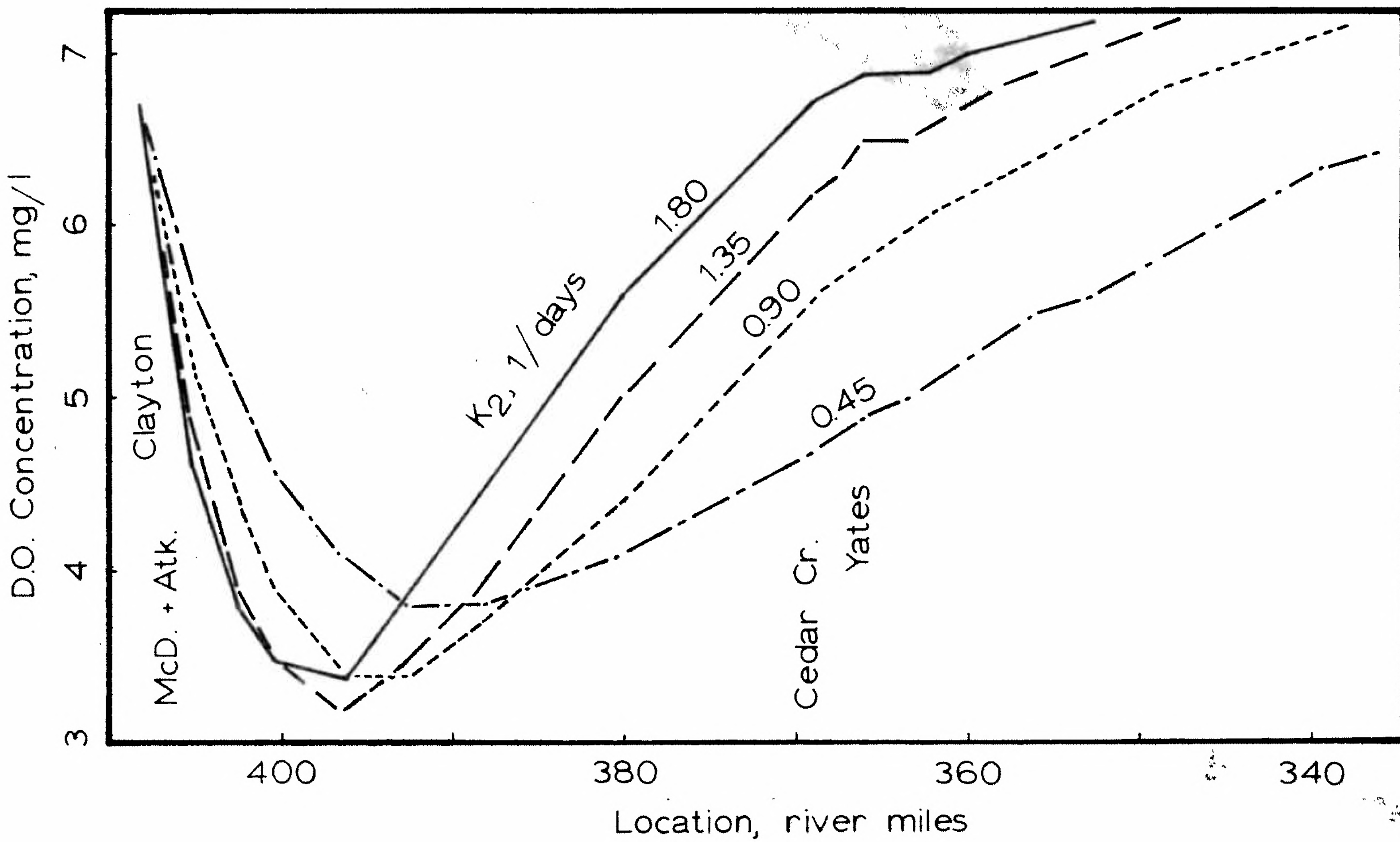

Figure 29. DO Profiles for K_2 Variation (September)

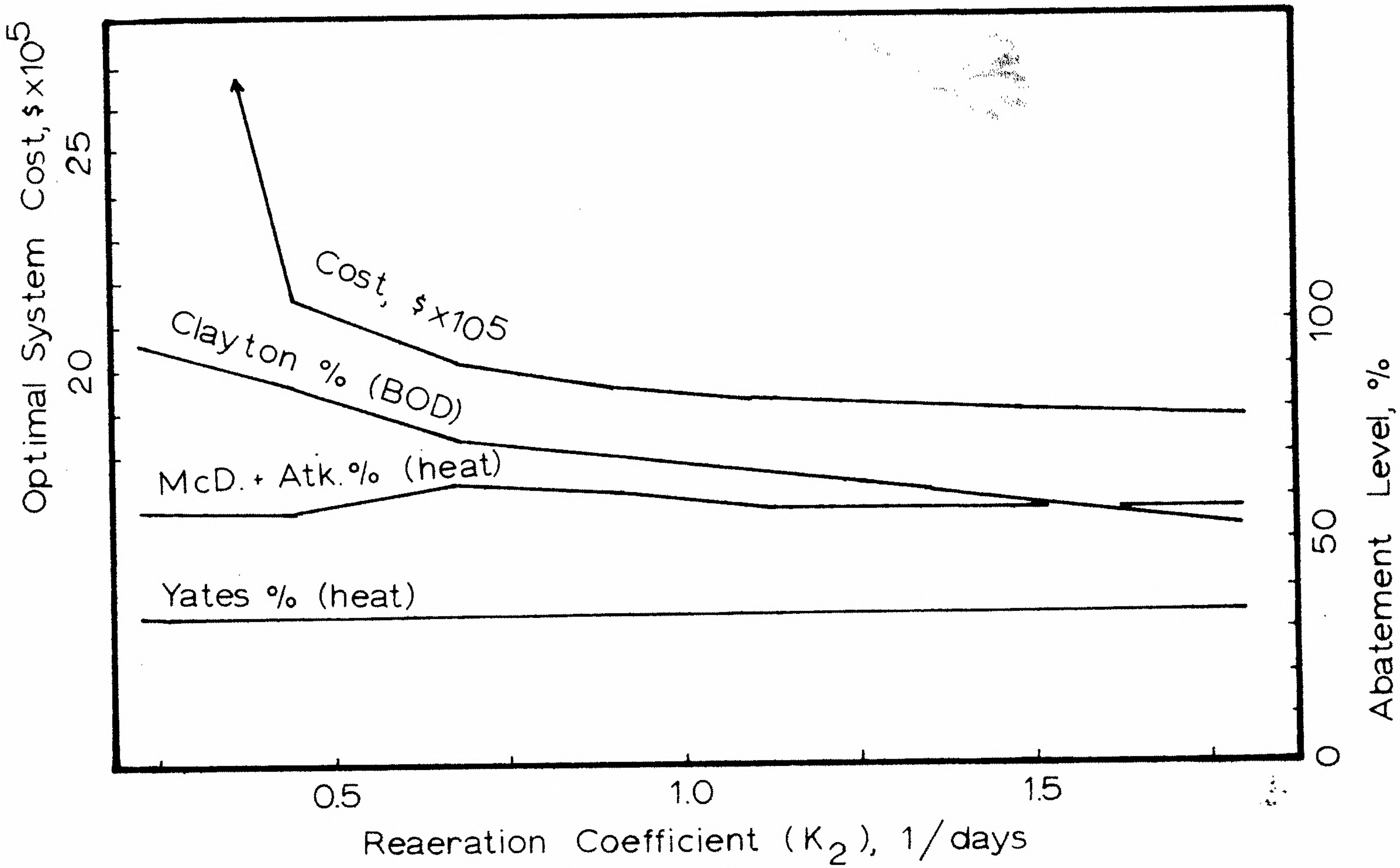

Figure 30. System Response to K_2 Variation (July)

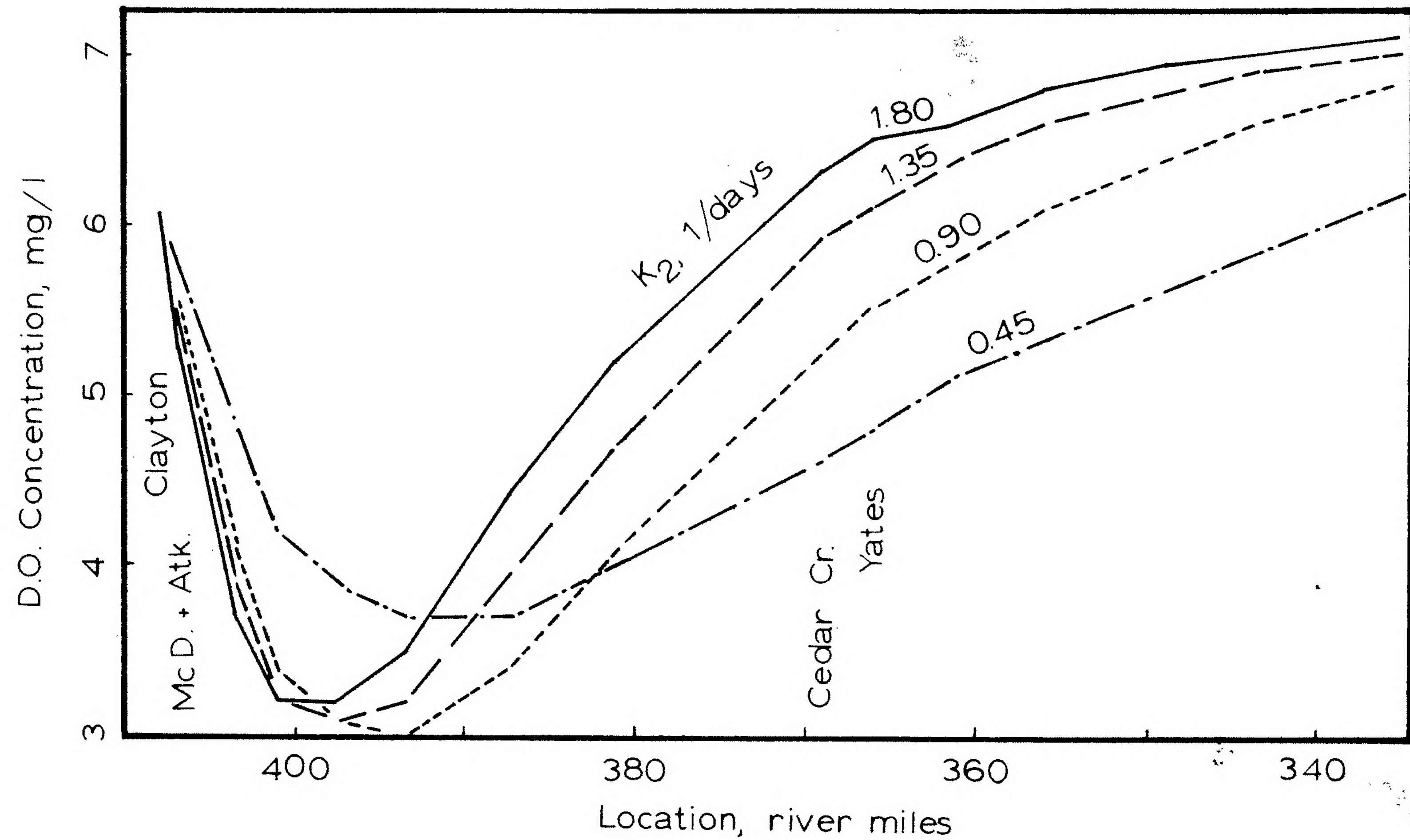

Figure 31. DO Profiles for K_2 Variation (July)

users are treating at levels prescribed by the optimal policy. If increased cooling decreased the Clayton level, this would occur when the Clayton treatment level exceeds 95 per cent. This is not the case, as no trade-off took place. The intermediate rise and fall of the McDonough and Atkinson cooling level is not thought to have any significance.

Variation of Water Quality Standards

It is apparent from earlier considerations that there is little or no interaction between thermal and organic pollutants in the Chattahoochee River basin as modeled in this investigation. This is attributed to the temperature standards used in this study which do not allow temperature levels to change enough to affect organic waste treatment. If a goal of the standards is to prevent such interaction, this has been attained when the standards are being satisfied. The economic effect of changes in the standards is unknown. It would be of interest to know the price being paid to prevent such pollutant interaction and the potential savings in abatement costs to be realized by less stringent standards. These savings could be compared to disbenefits which would accrue to other interests. It is also possible that public benefits are sufficient to justify even tighter standards. It is considered that inadequate effort has been devoted to a consideration of optimal water quality standards, and these thoughts motivated a consideration of the response of optimal system cost and abatement policy to a variation of DO and temperature standards.

Dissolved oxygen standards in all stages were varied from their present value plus 3 mg/ℓ to their present value minus 3 mg/ℓ in 1 mg/ℓ increments.* The temperature rise standard was varied from the present value of 10°F by plus and minus 6°F in 2°F increments, for a total of seven different standards. The maximum allowable temperature was varied in 2°F steps from its present value of 93°F by plus and minus 6°F. For each of these 343 sets of the three standards (DOS, TRS, and TMS), the optimal cost and abatement policy were determined. The system costs are presented in Figures 32, 33, 34, and 35.** The results for all values of TMS greater than 93°F are identical with the 93°F response. The reason for this is clear. The maximum possible unconstrained elevation in the system occurs at the McDonough-Atkinson outfall and is 12.3°F. An increase of 12.3°F to the equilibrium temperature of 80.7°F yields a maximum possible water temperature of 93°F. Therefore, relaxing the maximum temperature standards to above 93°F has no economic value, as no benefit results to any party.

Slight pollutant interaction was demonstrated for high values of DO standards (present level plus 3 mg/ℓ) and high values of the temperature rise standards (12°F and above) at a maximum temperature standard of 93°F or above. But since the maximum possible elevation is 12.3°F, this is of little consequence.

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*The highest set of DO standards was 6, 6, 7, and 7 mg/ℓ for stages 4 through 1, respectively; the lowest set, 0, 0, 1, and 1; a total of 7 sets.

**Resultant abatement policies are presented in Appendix B.

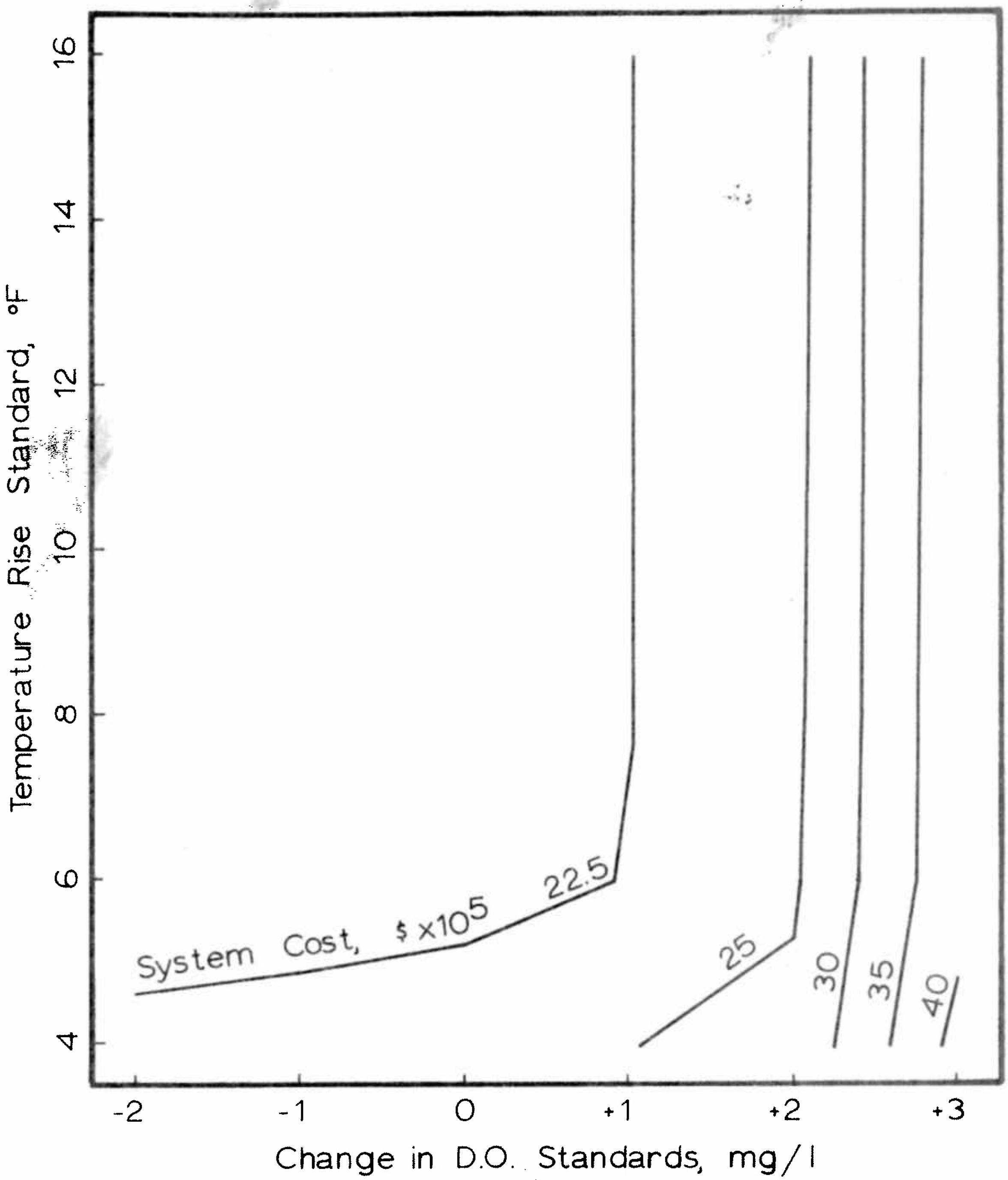

Figure 32. Optimal System Cost as a Function of Temperature Rise and Dissolved Oxygen Standards for a Maximum Temperature Standard of 87°F

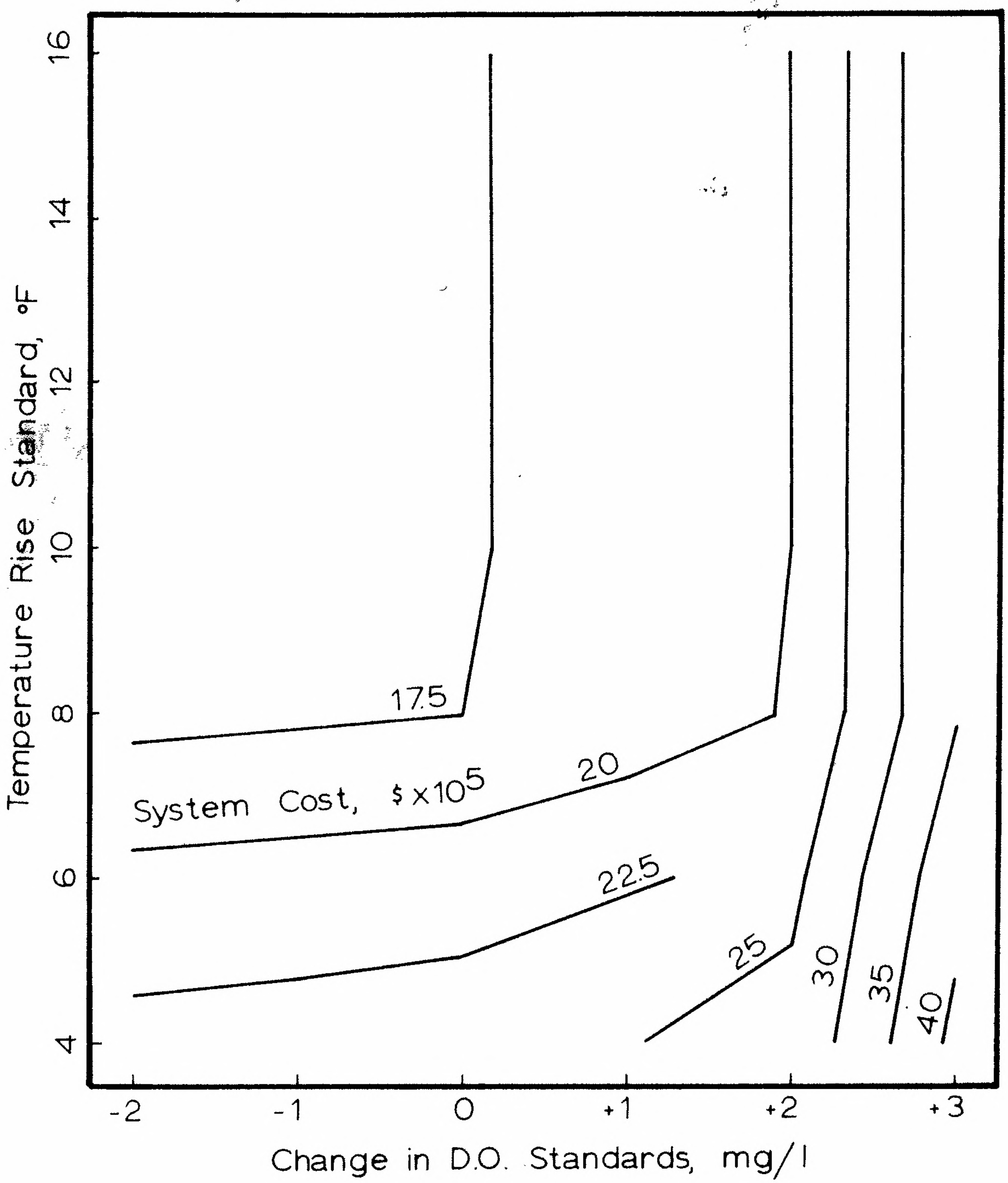

Figure 33. Optimal System Cost as a Function of Temperature Rise and Dissolved Oxygen Standards for a Maximum Temperature Standard of 89°F

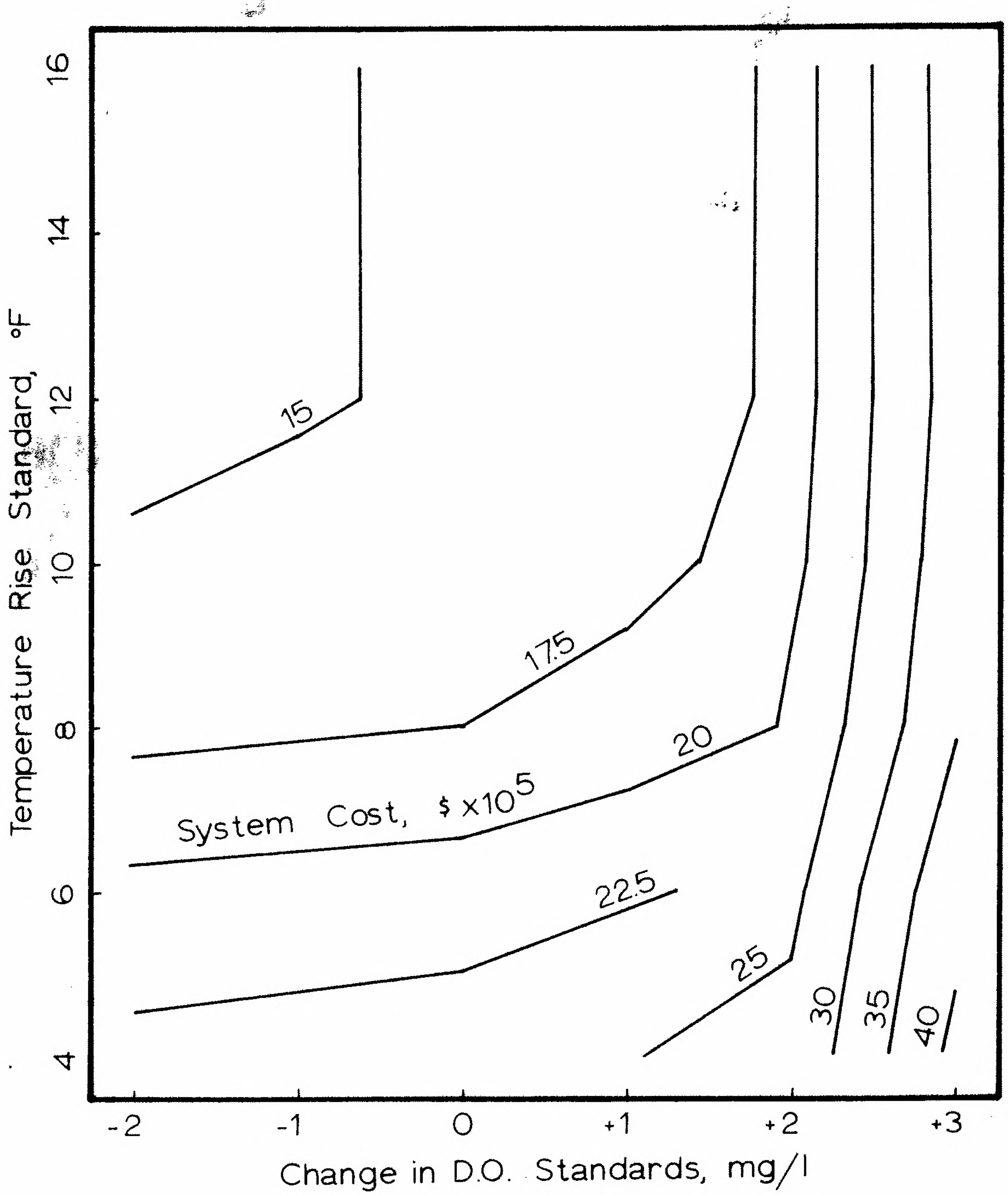

Figure 34. Optimal System Cost as a Function of Temperature Rise and Dissolved Oxygen Standards for a Maximum Temperature Standard of 91°F

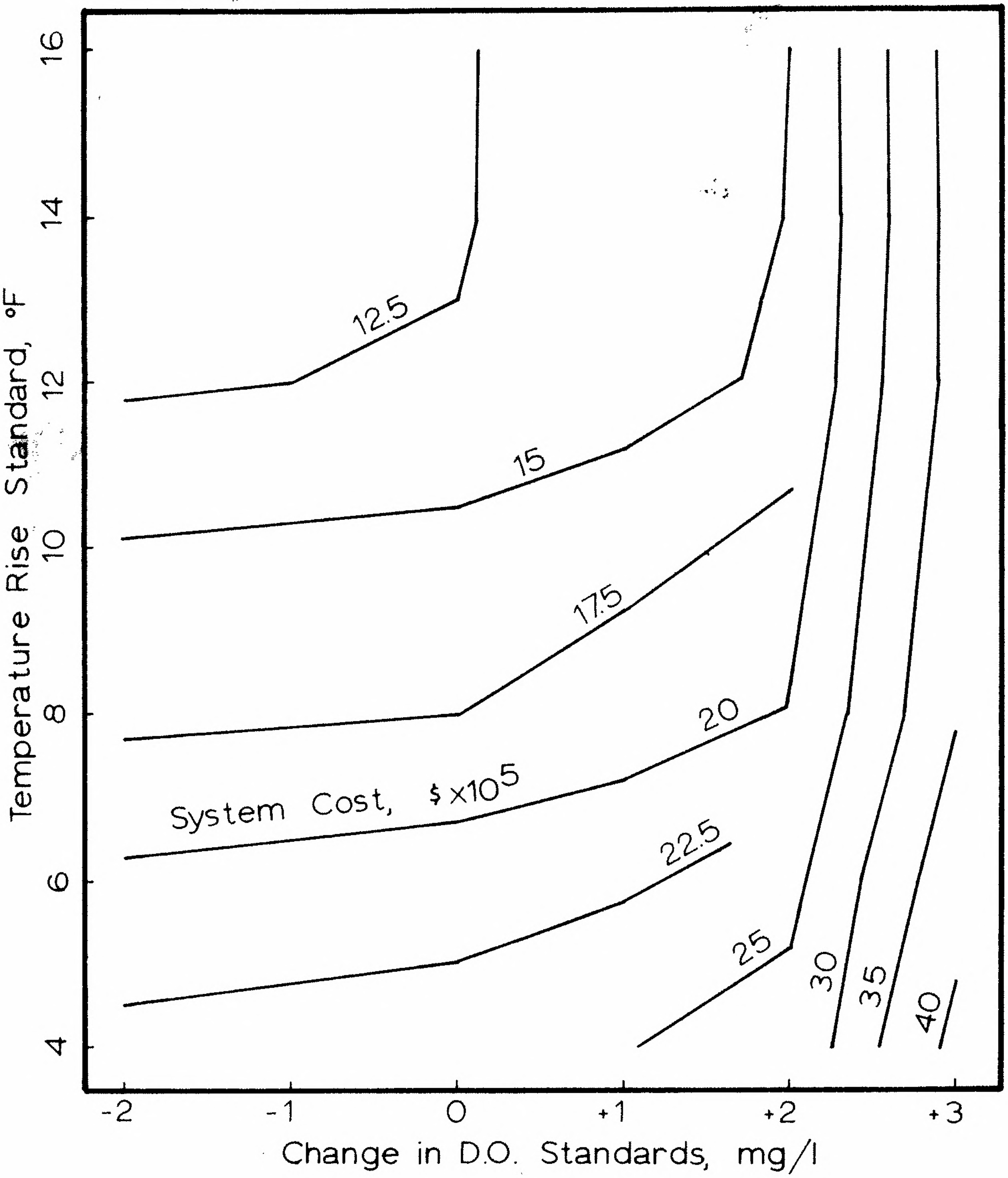

Figure 35. Optimal System Cost as a Function of Temperature Rise and Dissolved Oxygen Standards for a Maximum Temperature Standard of 93°F

The conclusion from the results presented in Figures 32, 33, 34, and 35 is that optimal system cost would be most affected by changes in dissolved oxygen standards. Very high DO values would probably be economically prohibitive. Decreases in temperature standards would be relatively inexpensive. Increases in temperature standards would result in only slight savings for abatement facilities.*

The advantage of this type of economic analysis is that alternative sets of water quality standards may be compared on a common, rational basis for cost effectiveness; and a more optimal management system may be instituted.

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*These remarks apply to the Chattahoochee River basin as modeled in this investigation. Extension to the general case is not justified, and extension to the real Chattahoochee River basin is questionable.

CHAPTER VIII

EFFECT OF AN ADDITIONAL HEAT SOURCE

The results of the previous chapter indicate that the Chattahoochee River basin would be unaffected by existing sources of thermal wastes, provided that cooling facilities were installed to allow the temperature standards to be met. Due to the fact that both existing sources of heat are located some distance from the critical DO zone, at about mile 394, it was decided to locate a hypothetical heat source at this point and observe the system response.

The waste heat production (2×10^9 Btu/hr) and resulting cooling cost were set arbitrarily between those for the existing sources. For September conditions, the streamflow and the rate coefficients (K_1 and K_2) were varied systematically as before, and optimal system cost and abatement policies were determined.

Variation of Streamflow

The results of the analysis for streamflow variation are shown in Figures 36 and 37. The system cost response curve is of the same general shape as that without the additional heat source (Figure 20) except that it appears to have been rotated clockwise about the right end. The +400 cfs cost value is the same as before, however the -300 cfs value is $500,000 greater. This is due entirely to the cooling cost for the new source. As before, the breaks in the curve at ±200 cfs are

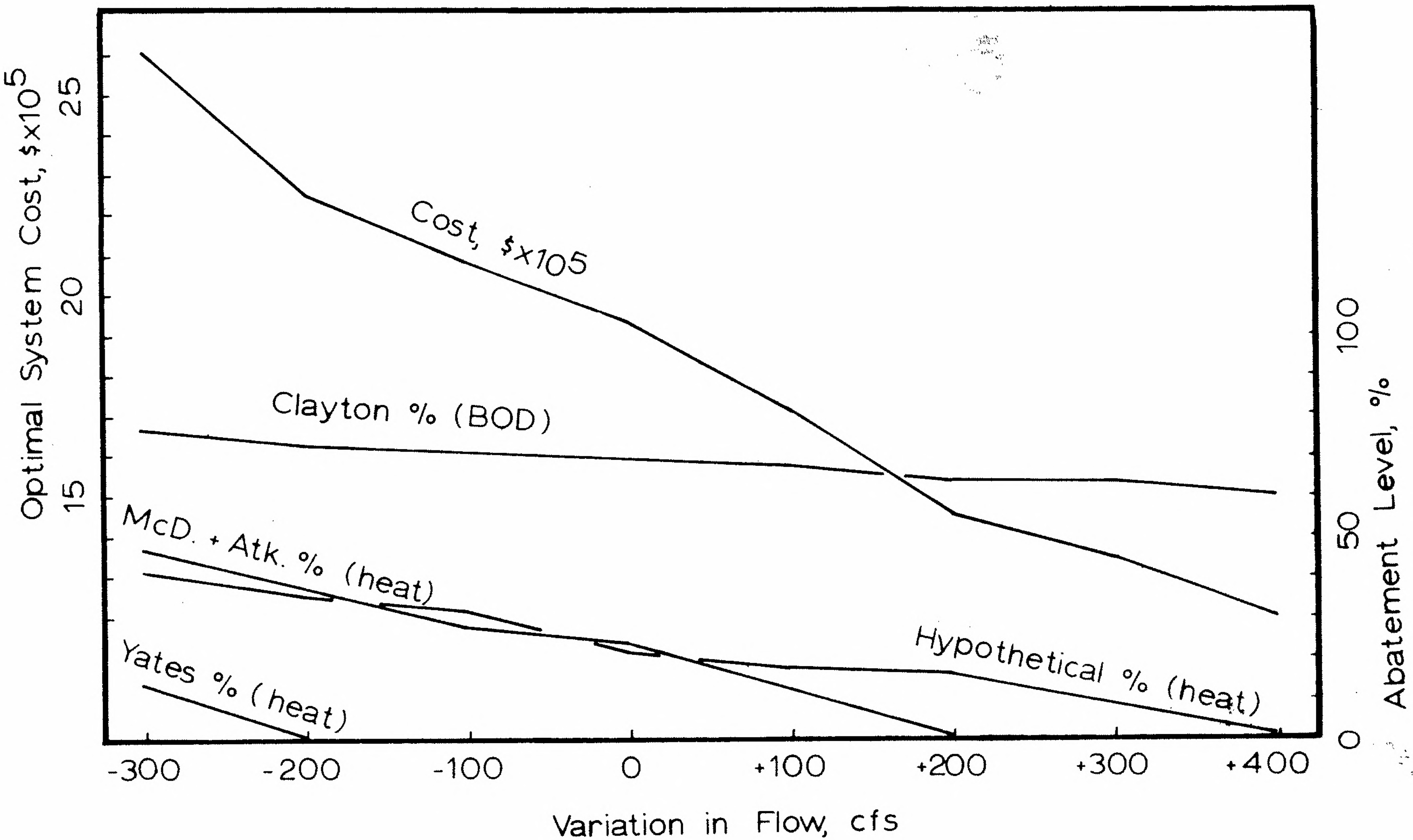

Figure 36. System Response to Flow Variation (September, Hypothetical)

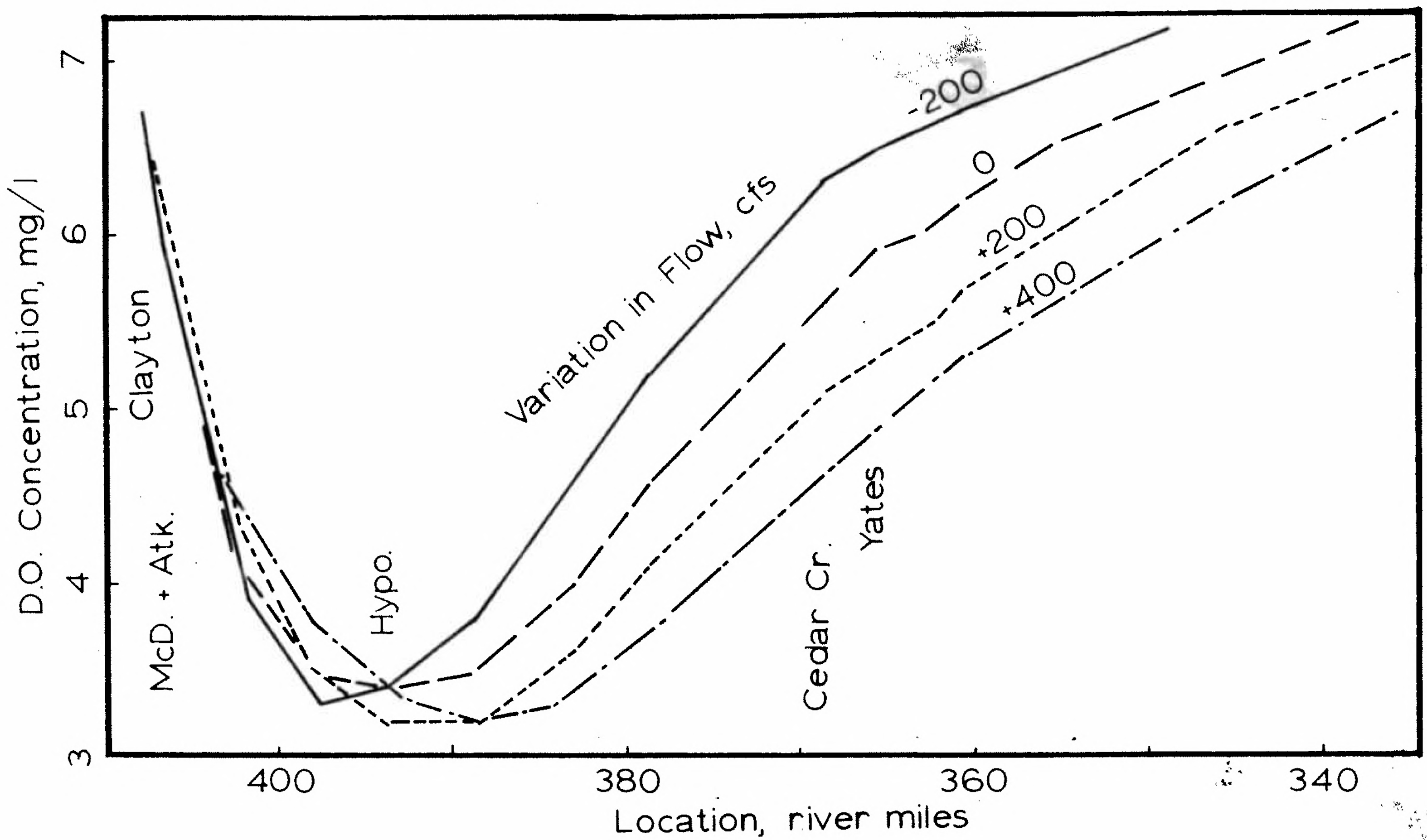

Figure 37. DO Profiles for Flow Variation (September, Hypothetical)

due to heat sources reaching a zero level of cooling and, therefore, creating no cost to the system.

The level of treatment at Clayton was not affected to a significant degree by the additional heat source. The Yates abatement level was unaffected, and only a mild interaction appeared between the hypothetical source and the McDonough and Atkinson heat source.

The conclusion is that the addition of a heat source in the Chattahoochee's critical DO zone would not cause any significant alteration of abatement levels at existing locations over a wide range of streamflows. A heat source between the critical zone and Clayton probably would have altered abatement levels to a greater extent.

Variation of the Deoxygenation Coefficient

The results of the analysis for variation of the deoxygenation coefficient, K_1, are shown in Figures 38 and 39. The system cost response to the additional heat source was a general rise (see Figure 24) due to increased cooling costs, being slightly more for low K_1 values. The effect on the Clayton treatment level was significant, raising required treatment by 40 per cent for the low K_1 value of 0.25. Intermediate values of K_1 resulted in a slight decrease in required treatment over conditions without the additional heat source. Consideration of Figure 39 indicates that this phenomena may be attributable, to some degree, to lower computer program resolution for runs at low K_1 values.

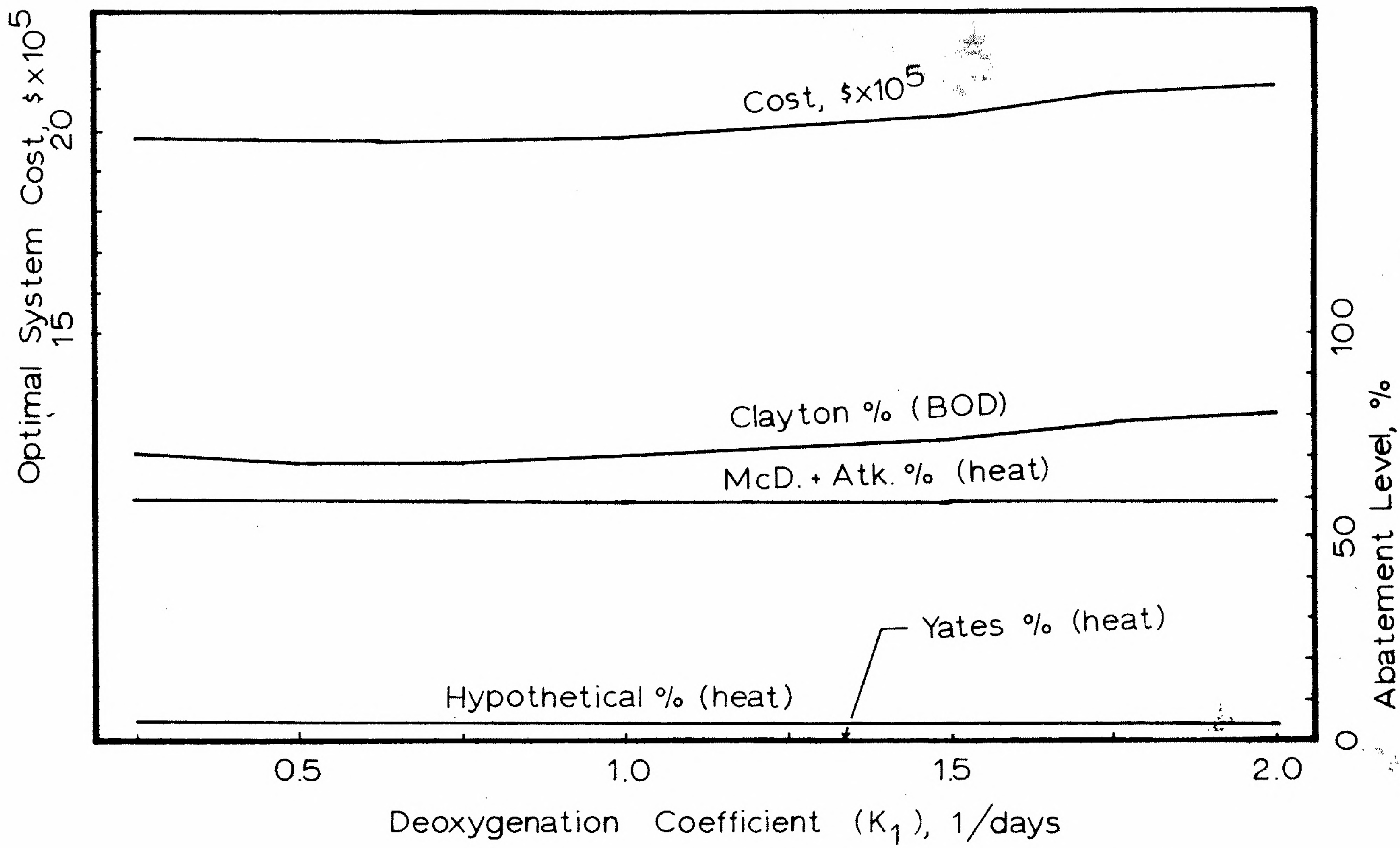

Figure 38. System Response to K_1 Variation (September, Hypothetical)

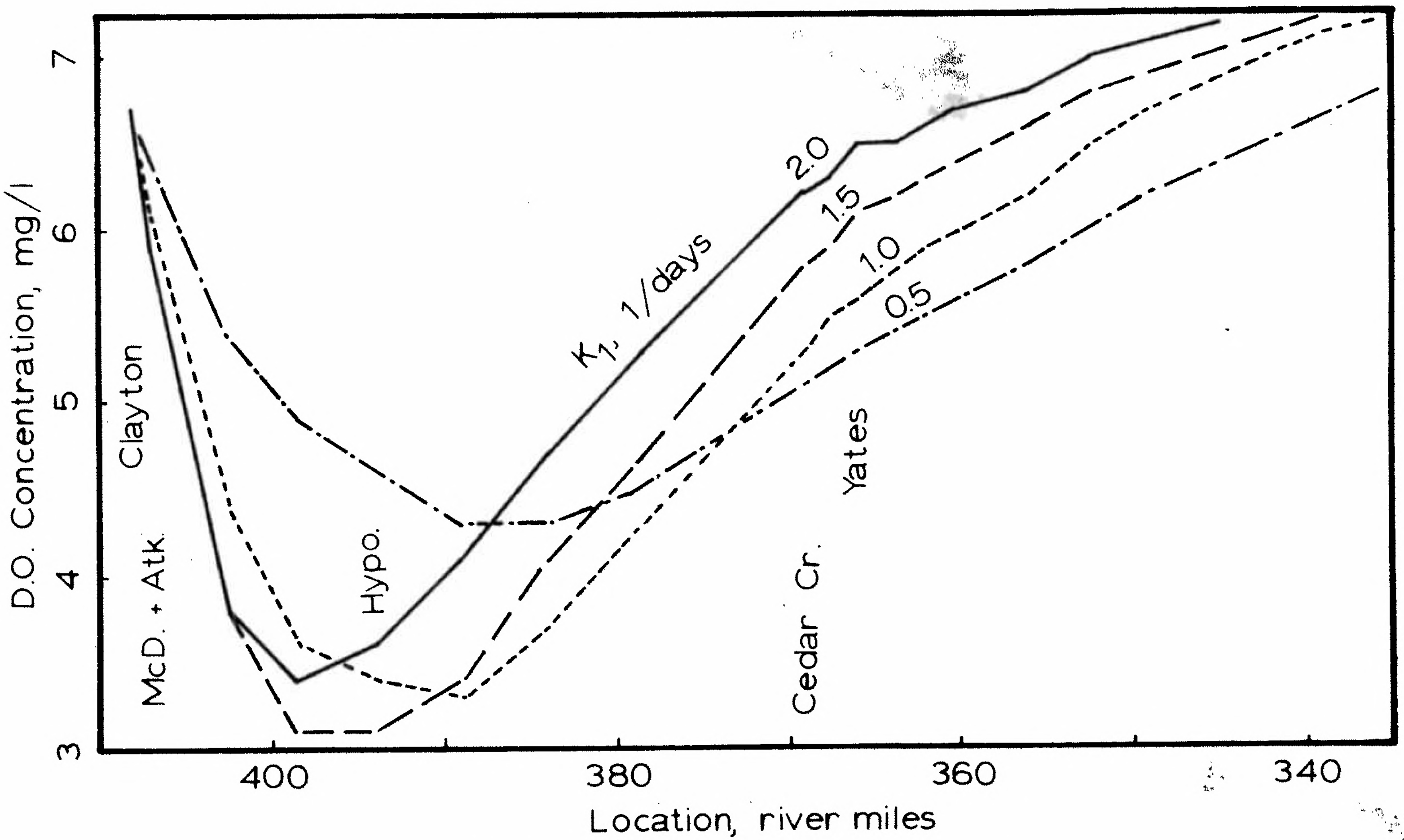

Figure 39. DO Profiles for K_1 Variation (September, Hypothetical)

The effect of the new heat source on cooling at other heat sources can be determined without difficulty and is quite significant. Whereas, under previous September conditions, McDonough and Atkinson (combined) provided 19 per cent cooling, the new heat source increased this to 59 per cent, while the additional source which brought this about provided only 4 per cent cooling in the optimal policy. If the management of the new source is different from that of McDonough and Atkinson, this situation is likely to result in the adoption of a non-optimal policy with regard to cooling. The cooling level at Yates remains at zero per cent.

Variation of the Reaeration Coefficient

Results of the analysis of optimal system cost and abatement policy response to variation of the reaeration coefficient, K_2, are shown in Figures 40 and 41. The general shape of the cost response is the same as without the additional heat source (see Figure 28). The cost is increased due to the additional heat source by about \$300,000 for a K_2 reduction of 75 per cent and increased by about \$400,000 for a 100 per cent increase of K_2. This is due principally to increased system cooling necessitated by the new heat source. The Clayton and Yates abatement levels remain essentially unchanged over conditions without the new source. Again, as with the variation of K_1, the new source provides only 4 per cent cooling and McDonough and Atkinson cooling is increased from 20 per cent to 56 per cent. The level of program resolution appears to be satisfactory for the K_2 variations as opposed to that for the K_1 variations from a consideration of Figure 41.

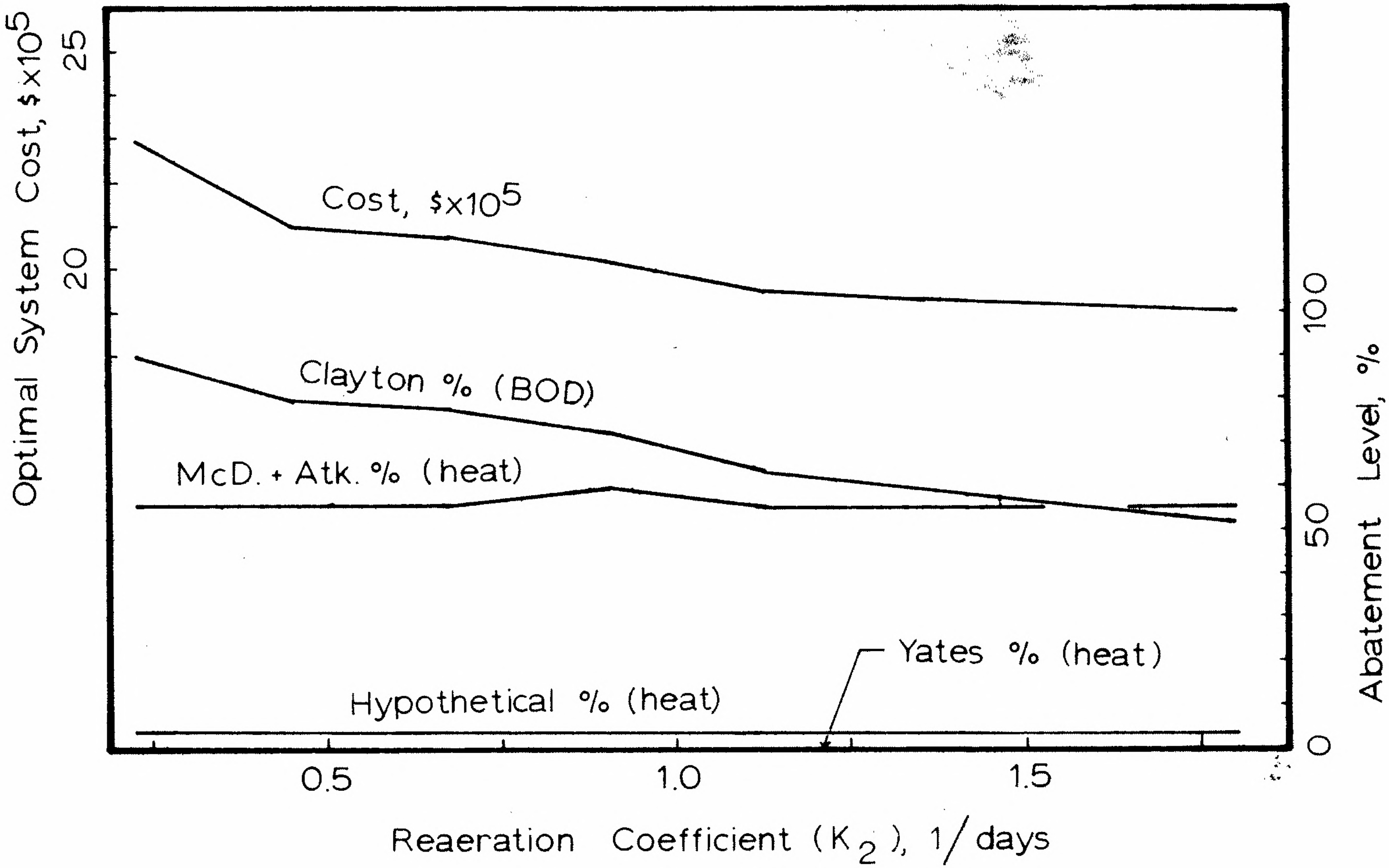

Figure 40. System Response to K_2 Variation (September, Hypothetical)

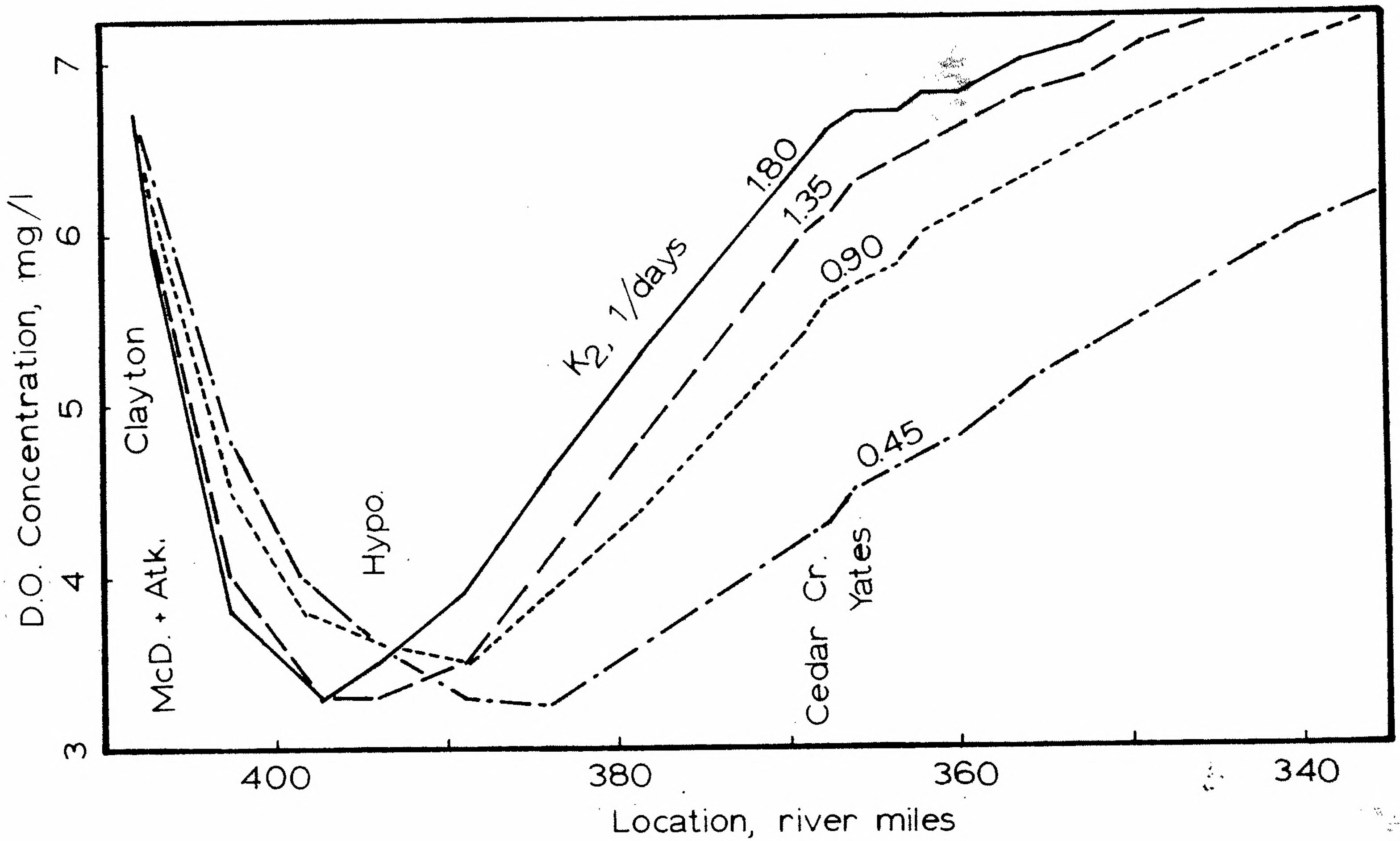

Figure 41. DO Profiles for K_2 Variation (September, Hypothetical)

In summary, the Chattahoochee River basin, as modeled in this investigation, seems to be very insensitive to the addition of waste heat for a wide range of important system parameters. A study such as that conducted in this chapter should be useful for investigating potential sites for the location of industries producing large amounts of thermal wastes. A significant degree of interaction between the industry and other producers of thermal or organic wastes could be demonstrated, and the increased costs for water quality maintenance considered along with other locational factors.

CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The purposes of this investigation have been (1) to develop methodology for an improved approach to regional water quality planning and management, (2) to use this methodology to demonstrate a procedure for minimizing the total abatement cost for thermal and organic wastes in a river basin while satisfying multiple stream standards, and (3) to demonstrate procedures for investigating system sensitivity to changes in quality standards and several important system parameters. These purposes have been fulfilled.

Although it is recognized that water quality planning and management activities are probably as responsive to social and political factors as they are to economic considerations, it is felt that the methodology developed in this investigation should be of considerable value to persons or agencies engaged in the planning and management of water quality. A knowledge of the minimum-cost abatement policy would provide an indication of the cost of attaining non-economic objectives in water resources development. Although the methodology developed in this investigation has been oriented specifically toward temperature and dissolved oxygen standards and thermal and organic wastes, the approach should be valid for the consideration of other sets of interacting wastes and their related water quality standards.

Given information regarding the hydrology, meteorology, and waste production for a river basin as outlined, the methodology presented herein may be used to determine the most economically efficient abatement policy for the basin. While the determination of such a policy for a single set of conditions is of much value, it is thought that additional insight and policy guidance may be obtained by a study of system sensitivity to the variation of water quality standards and important system parameters, such as streamflow and the deoxygenation and reaeration rate coefficients. For example, the results of an investigation of system response to changes in streamflow utilizing the methodology developed would allow an economically efficient combination of on-site abatement and flow augmentation to be devised. Also, an analysis of system response to a variation of water quality standards should put an agency on a firmer basis in the process of setting rational standards. The change in system cost can be compared to changes in public benefits for alternative sets of standards varying from low standards representing waste assimilation as the principal use of a stream to high standards representing a high level of recreation and aesthetics. In this manner, the best set of standards for the region may be selected. It is unlikely that the same standards would result in the best use of the water resources of all or most streams.

Although the principal benefits of the methodology developed in this investigation would be expected to accrue to regional or basin water quality planning agencies, the utility to industry is potentially great. The approach presented herein would be useful in locating appropriate sites for new or expanded production facilities. Severe interaction with existing waste producers could be determined at an early stage of the location process, and the higher abatement cost necessitated could be considered as an additional economic factor.

It is felt that the principal objective of this research has been attained; methodology for water quality planning and management which considers interacting wastes has been developed and demonstrated.

Recommendations

Most of the recommendations for future study are concerned with the use of the methodology developed in this investigation. Several of these recommendations stem from difficulties and deficiencies encountered in the Chattahoochee River basin example. This example should be accepted for what it is, a highly simplified representation of a complex real-world system that was used to demonstrate methodology.

It is recommended that the approach developed in this investigation be extended to consider conditions other than the steady-state with regard to waste inputs and streamflow. It is recognized that the use of the steady-state assumption greatly restricts the use of the approach.

Some attempt should be made to reconcile water quality standards with water resources data. For example, the use of instantaneous or mean daily standards with mean monthly waste and streamflow data has very serious limitations since the quality parameters will vary considerably in the stream over the longer period. To be of the greatest general benefit, however, planning methodology must make use of existing historical data, much of which is on a mean monthly basis.

One of the most important inputs which has a great influence on the regional abatement policy selected is cost data. The selection or assumption of a different cost-of-treatment relationship for a particular waste discharge would probably result in a significantly different optimal system cost and abatement policy. Reliable cost versus abatement level relationships are not presently available and are difficult to develop. This is an important area for continued study.

Increased precision may be obtained using the methodology developed herein by increasing the number of sub-reaches in each stage or stream reach. It would be desirable, however, to develop and use continuous transfer functions for the state variables instead of the discrete, step-wise approximations.

In all of the above considerations, one should realize that increasing precision and reliability result in increasing cost as well as data and computational requirements. It is necessary to guard against a desire for precision greater than that justified by the input information.

Computational costs would be unaffected by the time period used. This investigation was based on monthly data; other periods could be used if desired and if such data were available. Computational costs would increase directly with the number of sub-reaches in the stages or stream reaches. Such costs would increase exponentially with the number of stream reaches or stages used. Computational costs, as well as run time and core storage requirements, would also increase exponentially as the increment or step size on abatement levels and state variables are decreased.

In this investigation, methodology for water quality planning and management has been developed for cases where wastes interact; and its use for policy guidance has been demonstrated. Meaningful application to real-world river basins is left to governmental agencies and research groups that have adequate financial and personnel resources to obtain or to develop high quality input information.

One of the most pressing needs in the area of water quality planning and management is additional study of means to implement more efficient regional or basin management plans. From a consideration of the results of this investigation, as well as others of a similar nature in recent years, it appears that adequate capability exists to develop more efficient basin management policies and that the principal difficulty is in the institutional aspects of implementation. In a large part, this stems from greatly differing concepts of equity.

APPENDIX A

LISTING OF ALGOL PROGRAM FOR UNIVAC 1108 DIGITAL COMPUTER AND SAMPLE OUTPUT

```
          BEGIN

COMMENT             B.C.DYSART, GEORGIA INSTITUTE OF TECHNOLOGY
                    TWO-DIMENSIONAL DYNAMIC PROGRAMMING APPROACH TO REGIONAL
                    WATER QUALITY PLANNING AND MANAGEMENT WHICH MINIMIZES
                    ABATEMENT COSTS ASSOCIATED WITH THERMAL AND BIO-DEGRADABLE
                    ORGANIC WASTES WHILE MEETING SIMULTANEOUS TEMPERATURE AND
                    DISSOLVED OXYGEN CONSTRAINTS;

INTEGER             BLANKT,BLANKB,BLANKO,BLANKS,BLANKD,BLANK1,BLANK2,BLANKOS,
                    TBASE,S,TI,PASS,MAXPASS,I,J,K,L,M,N,P,JX,KX,LX,JJ,KK,LL,
                    MAXJ,MAXK,MAXL,COST,DCO,DTR,CMIN,NOR;
REAL                TRIS,TMIX;
REAL                FTIME,DPLOT,SPLOT,DPT,DPB,DPO,DPS,DPD,DP1,DP2,KON,
                    TEQ,DST,DSB,DSO,FORCE,TOUT,MINDOX,DA,LA,DT,DOX;
STRING ARRAY        LOC[20:0: 10];
FORMAT XLOC(A,S20);
ARRAY               TE,MT,MTC,K1,K2,SAT,H,W,O,DEF,BOD,DIOX[0:25];
FORMAT FXX(D6.2,A1.1);
FORMAT F01(E,'REACH =',I3,', Q =',I6,' CFS',A1,
          'T =',I12,' BTU/HR, TRS =',D5.1,', TMS =',D5.1,' DEG.F',A2,
          'B =',I12,' LB/DAY, DOS =',D5.1,', NSR =',I3,', K1 =',D5.2,
          ', K2 =',D5.2,A1,
          'STL =',D6.2,', STU =',D6.2,', DST =',D5.2,' DEG.F',A2,
          'SBL =',D6.2,', SBU =',D6.2,', DSB =',D5.2,' MG/L ',A1,
          'SOL =',D6.2,', SOU =',D6.2,', DSO =',D5.2,' MG/L ',A1,
          'CL  =',I6,', CU  =',I6,', DCO =',I5,' %',A1,
          'TL  =',I6,', TU  =',I6,', DTR =',I5,' %',A1,
          'COST  = XXX UNITS ',A2,
          'COOL  = XXX %     ',A1,
          'TREAT = XXX %     ',A1,
          'TS    = XXX DEG.F ',A1,
          'BS    = XXX MG/L  ',A1,
          'OS    = XXX MG/L  ',A1),
       F02(E,'REACH :',I3,'; INPUT TEMP.  = ST[I] =',D6.2,
                                   X15,'T[I] =',I12,'; B[I] =',I8,A1,
                         X12,'INPUT B.O.D. = SB[I] (DOWN LEFT SIDE)',A1,
                         X12,'INPUT D.O.   = SO[I] (ACROSS TOP)',A1,1),
       F05(E,'COST DATA :',A1.2),
       F06('REACH ',I2,A1),
       F07('%      ',21(I3,X1),A1),
       F08('COOL   ',21(I3,X1),A1),
       F09('TREAT ',21(I3,X1),A1.1),
       F10('UNIT TEMP. ELEVATION DATA :',A10.2),
       F13(E,'*****  OPTIMAL SCHEDULE  *****',A1.2,
          X6,'MIN. COST =',I9,A0,2,
          'RCH  *--- TEMP.IN,(ST) --*  *--- BOD IN,(SB) ---*  *-- DO ',
          'IN,(SO) --*  *--- COOL ---*  *-- TREAT ---*  *--- OUTPUT -',
          '--*',A,
          'NO.   MIN  DEG.F  MAX  DST   MIN   MG/L  MAX  DSB   MIN MG',
          '/L  MAX DSO  MIN  %  MAX DC  MIN  %  MAX DT  TEMP.  BOD  D',
          '.O.',A1,
          I2,X3,2(3(D5.2,X1),D3.1,X2),3(D4.2,X1),D3.1,X2,2(3(I3,X1),I2,
```

```
              X2),2(D5.2,X1),D4.2,A1),
         F14(I2,X3,2(3(D5.2,X1),D3.1,X2),3(D4.2,X1),D3.1,X2,2(3(I3,X1),I2,
              X2),2(D5.2,X1),D4.2,A1),
         F15(E,'***** PROFILES FOR OPTIMAL STRATEGY *****',A1.2,
              'REACH  TIME  TEMP  D.O.  DEF.  BOD.  TRIS  ***STANDARDS***',
              '*    K1     K2    SAT.',A,
              'NO.     FLOW  DEGF  ******MG/L******  DEGF  TMAX  TRIS  D.O',
              ',   1/DA   1/DA   MG/L',A1,
              X7,'DAYS',X32,'DEGF   DEGF   MG/L',A1.1),
         F16(X13,5(D4.1,X2),A1,I3,X4,D4.2,X32,3(D4.1,X2),3( D4.2,X2),A1),
         F17(X13,4(D4.1,X2),A1,I3,X4,D4.2,X50,3(D4.2,X2),A1),
         F18(X13,4(D4.1,X2),A1);
                     KON=0.000000044515669;
TI=CLOCK;
READ                 (NOR,TEQ,MAXPASS);
READ                 (DPT,DPB,DPO,DPLOT,TBASE);
READ                 (DPS,DP1,DP2,DPD);
                     FOR I=(0,1,NOR) DO
READ                 (XLOC,LOC[I]);
          BEGIN
ARRAY                STEPT,STEPB,STEPO,STEPCO,STEPTR[1:MAXPASS];
                     FOR PASS=(1,1,MAXPASS) DO
READ                 (STEPT[PASS],STEPB[PASS],STEPO[PASS],STEPCO[PASS],
                      STEPTR[PASS]);
          BEGIN
ARRAY                STL,STU,SBL,SBU,SOL,SOU,KD,KR,DOS,
                     ST,SB,SO,LODO,TRS,TMS[0:NOR],
                     TORD,TFS,PT,PB,PO,PS,PD,P1,P2[0:NOR,0:25];
INTEGER ARRAY        CL,CU,TL,TU,NSR,Q,T,B,COO,TRE,
                     LSTEP,JDIF,KDIF,LDIF[1:NOR],
                     CC,CT[1:NOR,0:100];
FORMAT FA6(E,' % ',:NOR:('  CC[',I2,'] CT[',I2,']'),A1.2),
       FA7(I3,X2,:NOR:(I5,X2,I5,X3),A1);
FORMAT XX1(E,X42,'P A R A M E T E R   P R O F I L E S',A1.1,
           X8,':',X39,'(TEMPERATURE, DEG.F.)',A1,
           X8,':',21(I2,X3),A1,
           X8,':',X34,'(DO, DOSAT, DODEF, & BOD, MG/L)',A1,
           ' FLOW  :00   01   02   03   04   05   06   07   08   09 ',
           ' 10   11   12   13   14   15   16   17   18   19   20',A1,
           ' TIME, :',X42,'(K1 & K2, 1/DAYS)',A1,
           ' DAYS  :00  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9 ',
           ' 1.0  1.1  1.2  1.3  1.4  1.5  1.6  1.7  1.8  1.9  2.0',A1,
           X8,':.    .    .    .    .    .    .    .    .    .    .   ',
           '  .    .    .    .    .    .    .    .    .    .    .',A1,
           '...............................................................',
           '..............................................................',
           A1.1);
FORMAT XX2(D7.3,' :',A);
FORMAT XX3(X9,:BLANKT:(X1),'T',A,X9,:BLANKB:(X1),'B',A,
           X9,:BLANKO:(X1),'O',A,X9,:BLANKS:(X1),'S',A,
           X9,:BLANK1:(X1),'1',A,X9,:BLANK2:(X1),'2',A,
           X9,:BLANKD:(X1),'D',A    );
FORMAT XX6(' ',A1);
FORMAT XX7(X8,':',:BLANKOS:(X1),'*',A1);
FORMAT XX13(X110,S20,A);
                     DOS[0]=DOS[1];
```

```
                FOR I=(1,1,NOR) DO
          BEGIN
READ            (T[I],B[I],Q[I],KD[I],KR[I],NSR[I],TRS[I],TMS[I],DOS[I]);
READ            (FOR M=(0,1,100) DO CC[I,M]);
READ            (FOR N=(0,1,100) DO CT[I,N]);
READ            (FOR P=(1,1,NSR[I]+1) DO TORD[I,P]);
READ            (FOR P=(1,1,NSR[I])    DO TFS [I,P]);
READ            (STL[I],STU[I],SBL[I],SBU[I],SOL[I],SOU[I],
                 CL[I],CU[I],TL[I],TU[I]);
          END;
                FOR PASS=(1,1,MAXPASS) DO
          BEGIN
                DST=STEPT [PASS];
                DSB=STEPB [PASS];
                DSO=STEPO [PASS];
                DCO=STEPCO[PASS];
                DTR=STEPTR[PASS];
                FOR I=(1,1,NOR) DO
          BEGIN
                JDIF[I]=(STU[I]-STL[I])/DST      ;
WRITE (FXX,JDIF[I]);
                KDIF[I]=(SBU[I]-SBL[I])/DSB      ;
WRITE (FXX,KDIF[I]);
                LDIF[I]=(SOU[I]-SOL[I])/DSO      ;
WRITE (FXX,LDIF[I]);
                LSTEP[I]=LDIF[I]+1;
                LODO[I]=MAX(DOS[I],DOS[I-1]);
          END;
                MAXJ=MAX(FOR I=(1,1,NOR) DO JDIF[I]);
WRITE (FXX,MAXJ);
                MAXK=MAX(FOR I=(1,1,NOR) DO KDIF[I]);
WRITE (FXX,MAXK);
                MAXL=MAX(FOR I=(1,1,NOR) DO LDIF[I]);
WRITE (FXX,MAXL);
          BEGIN

INTEGER ARRAY   BEST,COOL,TREAT[1:NOR,0:MAXJ,0:MAXK,0:MAXL];
ARRAY           TS,BS,OS        [1:NOR,0:MAXJ,0:MAXK,0:MAXL];

COMMENT         STEPPING ON REACHES;

                FOR I=(1,1,NOR) DO
          BEGIN

COMMENT         DETERMINING CORRESPONDENCE BETWEEN INDICES AND INPUTS;

                FOR J=(0,1,JDIF[I]) DO H[J]=J*DST+STL[I];
                FOR K=(0,1,KDIF[I]) DO W[K]=K*DSB+SBL[I];
                FOR L=(0,1,LDIF[I]) DO O[L]=L*DSO+SOL[I];

                IF PASS EQL MAXPASS THEN
WRITE           (F01,I,Q[I],T[I],TRS[I],TMS[I],B[I],DOS[I],NSR[I],KD[I],
                 KR[I],STL[I],STU[I],DST,SBL[I],SBU[I],DSB,SOL[I],SOU[I],
                 DSO,CL[I],CU[I],DCO,TL[I],TU[I],DTR);

COMMENT         STEPPING ON INPUT TEMPERATURE;
```

```
                FOR J=(0,1,JDIF[I]) DO
          BEGIN
                IF T[I] GTR 0 THEN CMIN=MAX(100-TRS[I]*Q[I]/(KON*T[I]),
                                   100-(TMS[I]-H[J])*Q[I]/(KON*T[I]));

COMMENT         STEPPING ON INPUT B.O.D.;

                FOR K=(0,1,KDIF[I]) DO
          BEGIN

COMMENT         STEPPING ON INPUT D.O.;

                FOR L=(0,1,LDIF[I]) DO
          BEGIN
                BEST(I,J,K,L)=999999;
                IF O[L] LSS DOS[I] THEN GO TO LODOIN;
COMMENT ******************************************************************;
COMMENT D.O. STDS AUTOMATICALLY VIOLATED, INCREMENT INCOMING D.O.;
COMMENT ******************************************************************;

COMMENT         STEPPING ON % COOLING;

                FOR M=(CL[I],DCO,CU[I]) DO
          BEGIN
                IF T[I] EQL 0 THEN BEGIN M=0; GO TO BYPASS; END;
                IF M LSS CMIN THEN GO TO MORECOOL;
COMMENT ******************************************************************;
COMMENT TEMP. STDS AUTOMATICALLY VIOLATED, INCREMENT COOL;
COMMENT ******************************************************************;
BYPASS:
                TRIS=KON*(100-M)*T[I]/Q[I];
                TMIX=H[J]+TRIS;
                FORCE=TMIX-TEQ;
                FOR P=(1,1,NSR[I]+1) DO
                TE[P]=TORD[I,P]*FORCE+TEQ;
                TOUT=TE[NSR[I]+1];
                FOR P=(1,1,NSR[I]) DO
          BEGIN
                MT[P]=(TE[P]+TE[P+1])/2;
                MTC[P]=5*(MT[P]-32)/9;
                K1[P]=KD[I]*1.047**(MTC[P]-20);
                K2[P]=KR[I]*1.024**(MTC[P]-20);
                SAT[P]=14.652-0.4102*MTC[P]+0.00799*MTC[P]**2-
                          0.0000777*MTC[P]**3;
          END;

COMMENT         STEPPING ON % TREATMENT;

                FOR N=(TL[I],DTR,TU[I]) DO
          BEGIN
                IF B[I] EQL 0 THEN N=0;
                MINDOX=20;
                IF SAT[1] LEQ O[L] THEN DA=0 ELSE DA=SAT[1]-O[L];
                LA=W[K]+0.001855*(100-N)*B[I]/Q[I];

COMMENT         STEPPING DOWN REACH CHECKING FOR D.O. VIOLATION;

                FOR P=(1,1,NSR[I]) DO
```

```
        BEGIN
                DT=K1[P]*LA/(K2[P]-K1[P])*(EXP(-K1[P]*TFS[I,P])-EXP
                   (-K2[P]*TFS[I,P]))+DA*EXP(-K2[P]*TFS[I,P]);
                DOX=SAT[P]-DT;
                IF DOX LSS DOS[I] AND B[I] EQL 0 AND T[I] EQL 0 THEN
                GO TO NEITHER;
COMMENT ***************************************************************;
COMMENT D.O. STDS VIOLATED, NO ABATEMENT HERE, MUST INCREMENT D.O. IN;
COMMENT ***************************************************************;
                IF DOX LSS DOS[I] AND B[I] EQL 0 THEN GO TO NOTREAT;
COMMENT ***************************************************************;
COMMENT WILL EXIT TREATMENT LOOP WITH TREAT = 0;
COMMENT ***************************************************************;
                IF DOX LSS DOS[I] THEN GO TO MORETREAT;
COMMENT ***************************************************************;
COMMENT D.O. TOO LOW INCREMENT TREAT;
COMMENT ***************************************************************;
                IF DOX LSS MINDOX THEN MINDOX=DOX;
                LA=LA*EXP(-K1[P]*TFS[I,P]);
                DA=SAT[P+1]-DOX;
        END OF STEPPING DOWN REACH LOOP;
                IF DOX LSS DOS[I-1] AND B[I] EQL 0 AND T[I] EQL 0 THEN
                GO TO NEITHER;
COMMENT ***************************************************************;
COMMENT D.O. LESS THAN D.O. STD IN REACH BELOW, INCREASE D.O. IN;
COMMENT ***************************************************************;
                IF DOX LSS DOS[I-1] AND B[I] EQL 0 THEN GO TO NOTREAT;
COMMENT ***************************************************************;
COMMENT OUTPUT D.O. LESS THAN STD IN REACH BELOW, MUST INCREASE D.O. IN;
COMMENT ***************************************************************;
                IF DOX LSS DOS[I-1] THEN GO TO MORETREAT;
COMMENT ***************************************************************;
COMMENT OUTPUT D.O. LESS THAN STD IN REACH BELOW, MUST INCREASE TREAT;
COMMENT ***************************************************************;
                COST=CC[I,M]+CT[I,N];
                IF I GTR 1 THEN
        BEGIN
                JX=(TOUT-STL[I-1])/DST;
                KX=(LA  -SBL[I-1])/DSB;
                LX=(DOX -SOL[I-1])/DSO;
                IF JX GTR JDIF[I-1] THEN
        BEGIN COST=888881; GO TO BOUNDED; END;
                IF KX GTR KDIF[I-1] THEN
        BEGIN COST=888882; GO TO BOUNDED; END;
                IF LX GTR LDIF[I-1] THEN
        BEGIN COST=888883; GO TO BOUNDED; END;
                IF JX LSS 0 THEN
        BEGIN COST=777771; GO TO BOUNDED; END;
                IF KX LSS 0 THEN
        BEGIN COST=777772; GO TO BOUNDED; END;
                IF LX LSS 0 THEN
        BEGIN COST=777773; GO TO BOUNDED; END;
                COST=COST+BEST[I-1,JX,KX,LX];
        END;
BOUNDED:
                IF COST LSS BEST[I,J,K,L] THEN
        BEGIN
                BEST [I,J,K,L]=COST;
```

```
                    COOL [I,J,K,L]=M;
                    TREAT[I,J,K,L]=N;
                    TS    [I,J,K,L]=TOUT;
                    BS    [I,J,K,L]=LA;
                    OS    [I,J,K,L]=DOX;
          END;
                    IF B[I] EQL 0 AND T[I] EQL 0 THEN GO TO NEITHER;
COMMENT ********************************************************************;
COMMENT SINCE NO WASTE, EXIT ABATEMENT LOOPS AT COOL & TREAT = 0;
COMMENT ********************************************************************;
                    IF B[I] EQL 0 THEN GO TO NOTREAT;
MORETREAT:

          END OF STEPPING ON % TREATMENT LOOP;

                    IF T[I] EQL 0 THEN GO TO NOCOOL;
COMMENT ********************************************************************;
COMMENT WILL EXIT COOLING LOOP WITH COOL = 0;
COMMENT ********************************************************************;
MORECOOL:
NOTREAT:

          END OF STEPPING ON % COOLING LOOP;

NOCOOL:
LODOIN:
NEITHER:

          END OF STEPPING ON INPUT D.O. LOOP;

          END OF STEPPING ON INPUT B.O.D. LOOP;

          END OF STEPPING ON INPUT TEMPERATURE LOOP;

                    IF PASS EQL MAXPASS THEN
          BEGIN
                    FOR J=(0,1,JDIF[I]) DO
          BEGIN

FORMAT F03(X5,:LSTEP[I]:(X4,D5.2),A1),
       FA3(D5.2,:LSTEP[I]:(X3,I6),A2),
       FB3(X5,:LSTEP[I]:(X6,I3),A1);
WRITE               (F02,I,H[J],T[I],B[I]);
WRITE               (F03,FOR L=(0,1,LDIF[I]) DO O[L]);
                    FOR K=(0,1,KDIF[I]) DO
          BEGIN
WRITE               (FA3,W[K],FOR L=(0,1,LDIF[I]) DO BEST [I,J,K,L]);
WRITE               (FB3,      FOR L=(0,1,LDIF[I]) DO COOL [I,J,K,L]);
WRITE               (FB3,      FOR L=(0,1,LDIF[I]) DO TREAT[I,J,K,L]);
WRITE               (F03,      FOR L=(0,1,LDIF[I]) DO TS    [I,J,K,L]);
WRITE               (F03,      FOR L=(0,1,LDIF[I]) DO BS    [I,J,K,L]);
WRITE               (F03,      FOR L=(0,1,LDIF[I]) DO OS    [I,J,K,L]);
          END;

          END;
          END;

          END OF STEPPING ON REACHES LOOP;
```

```
                IF PASS EQL MAXPASS THEN
          BEGIN
WRITE           (FA6,FOR I=(1,1,NOR) DO (I,I));
WRITE           (FA7,FOR M=(0,DCO,100) DO (M,FOR I=(1,1,NOR) DO
                   (CC[I,M],CT[I,M])));
WRITE           (F10);
                FOR I=(1,1,NOR) DO
          BEGIN

FORMAT F11('TFLOW    ',:NSR[I]:(D4.2,X1),A1),
       F12('ELEV.  ',:NSR[I]+1:(D4.2,X1),A1,1);
WRITE           (F06,I);
WRITE           (F11,FOR P=(1,1,NSR[I]) DO TFS[I,P]);
WRITE           (F12,FOR P=(1,1,NSR[I]+1) DO TORD[I,P]);

          END;
          END;
                I=NOR; J=0; K=0; L=0;
                COO[I]=COOL [I,J,K,L];
                TRE[I]=TREAT[I,J,K,L];
WRITE           (F13,BEST[I,J,K,L],I,STL[I],STL[I],STU[I],DST,
                    SBL[I],SBL[I],SBU[I],DSB,SOL[I],SOL[I],SOU[I],DSO,
                    CL[I],COO[I],CU[I],DCO,TL[I],TRE[I],TU[I],DTR,
                    TS[I,J,K,L],BS[I,J,K,L],OS[I,J,K,L]);
                JJ=(TS[I,J,K,L]-STL[I-1])/DST;
                KK=(BS[I,J,K,L]-SBL[I-1])/DSB;
                LL=(OS[I,J,K,L]-SOL[I-1])/DSO;
                FOR I=(NOR-1,-1,1) DO
          BEGIN
                J=JJ; K=KK; L=LL;
                COO[I]=COOL [I,J,K,L];
                TRE[I]=TREAT[I,J,K,L];
                ST[I]=J*DST+STL[I];
                SB[I]=K*DSB+SBL[I];
                SO[I]=L*DSO+SOL[I];
WRITE           (F14,          I,STL[I],ST [I],STU[I],DST,
                    SBL[I],SB [I],SBU[I],DSB,SOL[I],SO [I],SOU[I],DSO,
                    CL[I],COO[I],CU[I],DCO,TL[I],TRE[I],TU[I],DTR,
                    TS[I,J,K,L],BS[I,J,K,L],OS[I,J,K,L]);
                JJ=(TS[I,J,K,L]-STL[I-1])/DST;
                KK=(BS[I,J,K,L]-SBL[I-1])/DSB;
                LL=(OS[I,J,K,L]-SOL[I-1])/DSO;
          END;
WRITE ('P A S S  =',PASS);

          END;
WRITE           (F15);
                FOR I=(NOR,-1,1) DO
          BEGIN
                M=COO[I];
                N=TRE[I];
                ST[I]=IF I EQL NOR THEN STL[I] ELSE TOUT;
                SB[I]=IF I EQL NOR THEN SBL[I] ELSE LA*Q[I+1]/Q[I];
                SO[I]=IF I EQL NOR THEN SOL[I] ELSE DOX;
                TRIS=KON*(100-M)*T[I]/Q[I];
                TMIX=ST[I]+TRIS;
                FORCE=TMIX-TEQ;
```

```
                FOR P=(1,1,NSR[I]+1) DO
                       TE[P]=TORD[I,P]*FORCE+TEQ;
                TOUT=TE[NSR[I]+1];
                FOR P=(1,1,NSR[I]) DO
           BEGIN
                MT[P]=(TE[P]+TE[P+1])/2;
                MTC[P]=5*(MT[P]-32)/9;
                K1[P]=KD[I]*1.047**(MTC[P]-20);
                K2[P]=KR[I]*1.024**(MTC[P]-20);
                SAT[P]=14.652-0.4102*MTC[P]+0.00799*MTC[P]**2-
                      0.0000777*MTC[P]**3;
           END;
                DEF[1]=DA=IF SAT[1] GTR SO[I] THEN SAT[1]-SO[I] ELSE 0;
                       DIOX[1]=IF SAT[1] GTR SO[I] THEN SO[I]
                       ELSE SAT[1];
                       BOD[1]=LA=SB[I]+0.001855*(100-N)*B[I]/Q[I];
                FOR P=(1,1,NSR[I]) DO
           BEGIN
                DEF[P+1]=K1[P]*LA/(K2[P]-K1[P])*(EXP(-K1[P]*TFS[I,P])-
                        EXP(-K2[P]*TFS[I,P]))+DA*EXP(-K2[P]*TFS[I,P]);
                         DOX=DIOX[P+1]=SAT[P]-DEF[P+1];
                         LA=BOD[P+1]=LA*EXP(-K1[P]*TFS[I,P]);
                DA=SAT[P+1]-DOX;
           END;
WRITE           (F16,TE[1],DIOX[1],DEF[1],BOD[1],TRIS,I,TFS[I,1],TMS[I],
                 TRS[I],DOS[I],K1[1],K2[1],SAT[1]);
                FOR P=(2,1,NSR[I]) DO
WRITE           (F17,TE[P],DIOX[P],DEF[P],BOD[P],I,TFS[I,P],K1[P],K2[P],
                 SAT[P]);
                FOR P=NSR[I]+1 DO
WRITE           (F18,TE[P],DIOX[P],DEF[P],BOD[P]);
                FOR P=(1,1,NSR[I]+1) DO
           BEGIN
                PT[I,P]=TE[P];
                PB[I,P]=BOD[P];
                PO[I,P]=DIOX[P];
                PS[I,P]=SAT[P];
                P1[I,P]=K1 [P];
                P2[I,P]=K2 [P];
                PD[I,P]=DEF[P];
           END;
           END;
COMMENT ***************************************************************;
COMMENT         PLOTTING PARAMETER PROFILES VERSUS TIME-OF-FLOW;
COMMENT ***************************************************************;
                FTIME=0;
WRITE           (XX1,FOR I=(TBASE,1,TBASE+20) DO I);
                I=NOR;
L3:
                P=1;
L2:
WRITE           (XX2,FTIME);
                BLANKOS=DOS[I]/DPO;
                BLANKT=(PT[I,P]-TBASE)/DPT;
                BLANKB=PB[I,P]/DPB;
                BLANKO=PO[I,P]/DPO;
                BLANKS=PS[I,P]/DPS;
                BLANK1=P1[I,P]/DP1;
```

```
                   BLANK2=P2[I,P]/DP2;
                   BLANKD=PD[I,P]/DPD;
WRITE              (XX3);
                   IF P EQL 1 THEN
WRITE              (XX13,LOC[I]);
                   IF I EQL 1 AND P EQL NSR[I]+1 THEN
WRITE              (XX13,LOC[0]);
                   IF P LSS NSR[I]+1 THEN
            BEGIN
                   SPLOT=TFS[I,P]/DPLOT;
                   FOR S=(1,1,SPLOT-1) DO
            BEGIN
WRITE              (XX7);
            END;
WRITE              (XX6);
                   FTIME=FTIME+TFS[I,P];
                   P=P+1;
                   GO TO L2;
            END;
                   IF I GTR 1 THEN
            BEGIN
                   I=I-1;
                   GO TO L3;
            END;
                   IF PASS LSS MAXPASS THEN
            BEGIN
                   FOR I=(1,1,NOR-1) DO
            BEGIN
                   STL[I]=MAX(ST[I]-  DST,TEQ);
                   STU[I]=ST[I]+  DST;
                   SBL[I]=MAX(SB[I]-3*DSB,0);
                   SBU[I]=SB[I]+3*DSB;
                   SOL[I]=MAX(SO[I]-DSO,DOS[I]);
                   SOU[I]=SO[I]+  DSO;
            END;
            END;
            END;

            END;

            END;

            END OF PROGRAM;
```

Table 11. Summary Output of Typical Run Indicating Optimal System Cost, Abatement Policy ($\bar{D}^*$), Stage Inputs ($\bar{S}_n$), Resolution, and Profiles of System Parameters for the Optimal Abatement Policy

***** OPTIMAL SCHEDULE *****

MIN. COST = 1576

RCH	*--- TEMP.IN,(ST) ---*				*--- BOD IN,(SB) ---*				*-- DO IN,(SO) --*			
NO.	MIN	DEG.F	MAX	DST	MIN	MG/L	MAX	DSB	MIN	MG/L	MAX	DSO
4	80.70	80.70	80.70	.5	2.00	2.00	2.00	.5	6.70	6.70	6.70	.5
3	80.70	80.70	81.20	.5	4.89	8.89	10.89	.5	5.68	6.18	6.68	.5
2	81.08	81.58	82.08	.5	0.00	.50	3.45	.5	5.06	5.56	6.06	.5
1	80.93	81.43	81.93	.5	0.00	.50	3.34	.5	5.35	5.85	6.35	.5

--- COOL ---				*-- TREAT ---*				*--- OUTPUT ---*		
MIN	%	MAX	DC	MIN	%	MAX	DT	TEMP.	BOD	D.O.
0	0	0	2	0	70	100	2	80.70	8.83	6.14
0	20	50	2	0	0	0	2	81.58	.51	5.32
0	0	0	2	0	0	0	2	81.43	.39	5.84
0	0	20	2	0	0	0	2	81.04	.01	7.53

***** PROFILES FOR OPTIMAL STRATEGY *****

REACH	TIME	TEMP	D.O.	DEF.	BOD.	TRIS	***STANDARDS****			K1	K2	SAT.
NO.	FLOW DAYS	DEGF	******MG/L******			DEGF	TMAX DEGF	TRIS DEGF	D.O. MG/L	1/DA	1/DA	MG/L
		80.7	6.7	1.2	9.5	0.0						
4	.05						93.0	10.0	3.0	1.38	1.06	7.86
		80.7	6.1	1.7	8.8							
		90.5	6.1	1.0	8.3	9.8						
3	.10						93.0	10.0	3.0	1.75	1.20	7.13
		89.4	5.0	2.1	7.0							
3	.10									1.71	1.19	7.21
		88.5	4.2	3.0	5.9							
3	.10									1.66	1.17	7.29
		87.4	3.7	3.6	5.0							
3	.20									1.61	1.15	7.39
		85.9	3.2	4.1	3.6							
3	.20									1.56	1.13	7.49
		84.8	3.2	4.2	2.6							
3	.20									1.52	1.12	7.57
		83.8	3.5	4.1	2.0							
3	.40									1.47	1.10	7.66
		82.6	4.3	3.4	1.1							
3	.57									1.43	1.08	7.75
		81.6	5.4	2.3	.5							
		81.6	5.4	2.3	.5	0.0						
2	.07						93.0	10.0	4.0	1.41	1.08	7.79
		81.5	5.6	2.2	.4							
2	.10									1.41	1.07	7.80
		81.4	5.7	2.1	.4							
		89.2	5.7	1.5	.4	7.7						
1	.10						93.0	10.0	4.0	1.69	1.18	7.24
		88.1	5.9	1.4	.3							
1	.10									1.65	1.17	7.31
		87.1	6.0	1.3	.3							
1	.10									1.61	1.15	7.38
		86.4	6.1	1.3	.2							
1	.20									1.57	1.14	7.46
		85.1	6.3	1.1	.2							
1	.20									1.53	1.12	7.55
		84.1	6.5	1.0	.1							
1	.20									1.49	1.11	7.62
		83.3	6.7	.9	.1							
1	.50									1.45	1.09	7.70
		82.1	7.1	.6	.0							
1	.50									1.42	1.08	7.78
		81.4	7.4	.4	.0							
1	.50									1.40	1.07	7.82
		81.0	7.5	.3	.0							

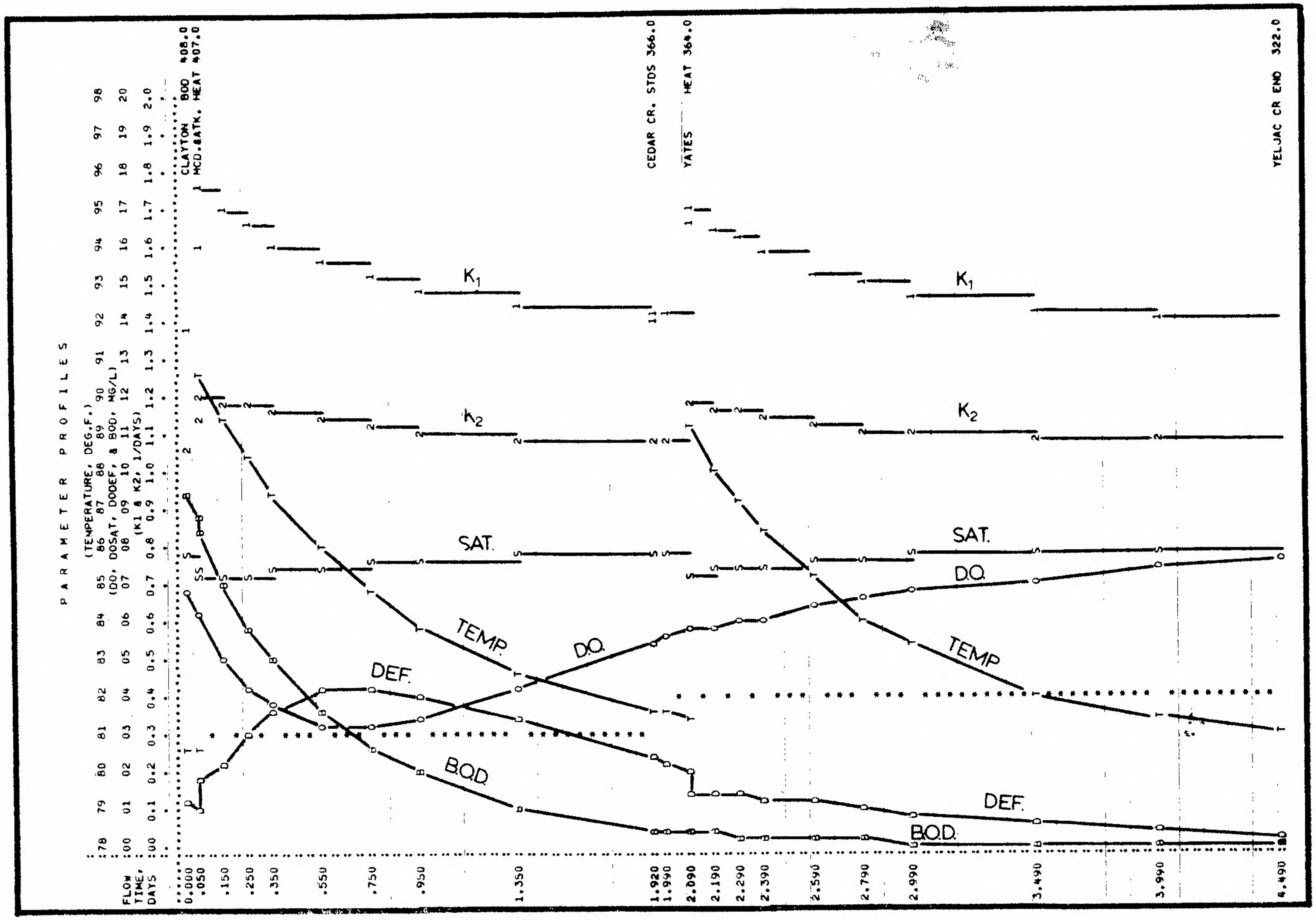

Figure 42. Plot of System Parameter Profiles for Optimal Abatement Policy of Table 11

APPENDIX B

OPTIMAL SYSTEM COST AND ABATEMENT POLICY AS A FUNCTION OF TEMPERATURE RISE (TRS) AND DISSOLVED OXYGEN (DOS) STANDARDS FOR VALUES OF MAXIMUM TEMPERATURE (TMS) STANDARD VARYING FROM 87°F TO 93°F

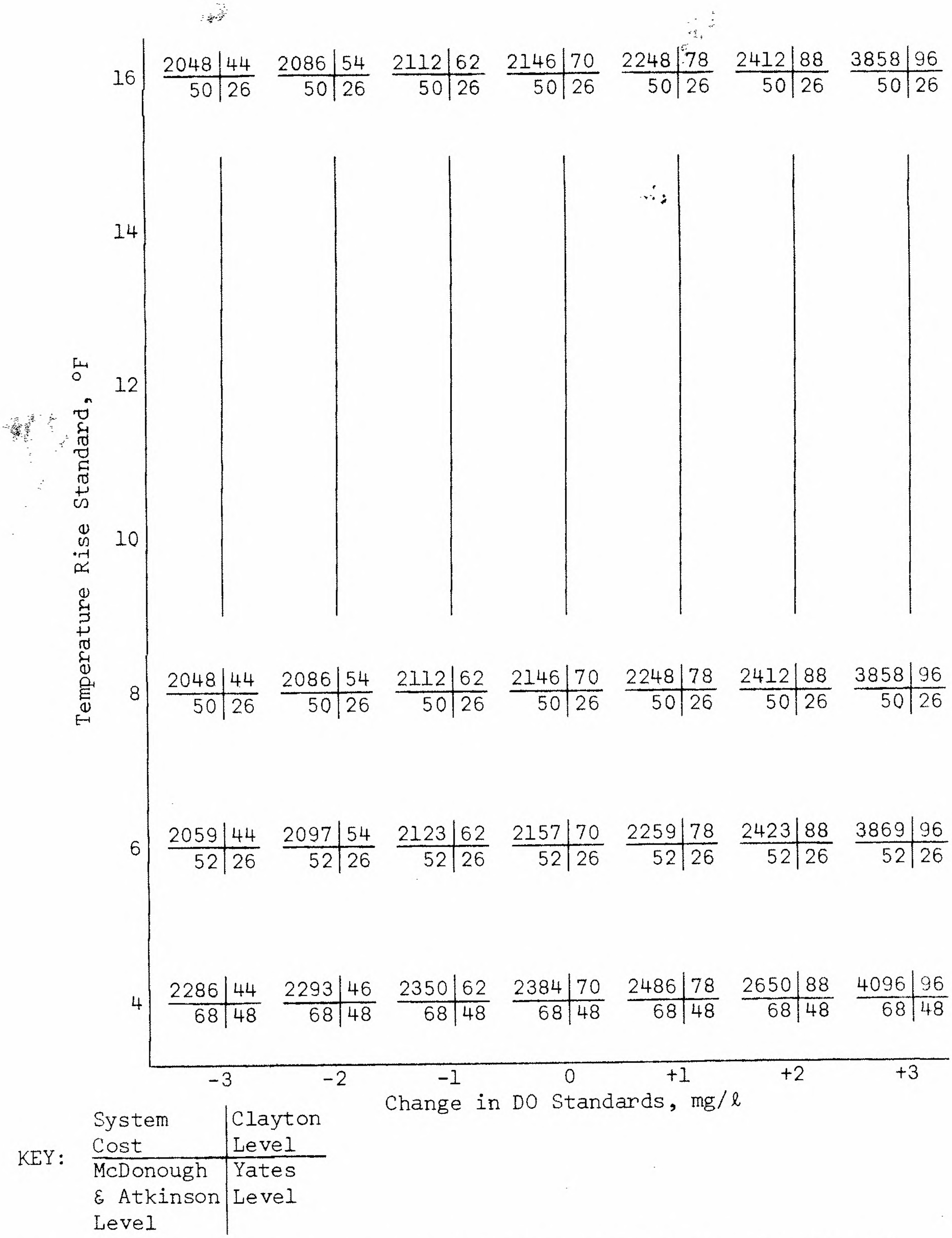

Figure 43. Optimal System Cost and Abatement Policy as a Function of Temperature Rise and Dissolved Oxygen Standards for a Maximum Temperature Standard of 87°F.

Temperature Rise Standard, °F (rows) vs. Change in DO Standards, mg/ℓ (columns). Each cell: System Cost, Clayton Level; McDonough & Atkinson Level, Yates Level.

Temperature Rise Standard, °F	-3	-2	-1	0	+1	+2	+3
16	1635, 44; 34, 0	1673, 54; 34, 0	1669, 62; 34, 0	1733, 70; 34, 0	1835, 78; 34, 0	1999, 88; 34, 0	3445, 96; 34, 0
14							
12							
10	1635, 44; 34, 0	1673, 54; 34, 0	1669, 62; 34, 0	1733, 70; 34, 0	1835, 78; 34, 0	1999, 88; 34, 0	3445, 96; 34, 0
8	1653, 44; 36, 0	1691, 54; 36, 0	1717, 62; 36, 0	1751, 70; 36, 0	1753, 78; 36, 0	2017, 88; 36, 0	3463, 96; 36, 0
6	2025, 44; 52, 22	2063, 54; 52, 22	2089, 62; 52, 22	2123, 70; 52, 22	2225, 78; 52, 22	2389, 88; 52, 22	3835, 96; 52, 22
4	2286, 44; 68, 48	2293, 46; 68, 48	2350, 62; 68, 48	2384, 70; 68, 48	2486, 78; 68, 48	2650, 88; 68, 48	4096, 96; 68, 48

Change in DO Standards, mg/ℓ

KEY:

System Cost	Clayton Level
McDonough & Atkinson Level	Yates Level

Figure 44. Optimal System Cost and Abatement Policy as a Function of TRS and DOS for TMS = 89°F

Temperature Rise Standard, °F	-3	-2	-1	0	+1	+2	+3
16	1423 | 44 / 16 | 0	1461 | 54 / 16 | 0	1487 | 62 / 16 | 0	1521 | 70 / 16 | 0	1623 | 78 / 16 | 0	1787 | 88 / 16 | 0	3233 | 96 / 16 | 0
14							
12	1423 | 44 / 16 | 0	1461 | 54 / 16 | 0	1487 | 62 / 16 | 0	1521 | 70 / 16 | 0	1623 | 78 / 16 | 0	1787 | 88 / 16 | 0	3233 | 96 / 16 | 0
10	1478 | 44 / 20 | 0	1516 | 54 / 20 | 0	1542 | 62 / 20 | 0	1576 | 70 / 20 | 0	1678 | 78 / 20 | 0	1842 | 88 / 20 | 0	3288 | 96 / 20 | 0
8	1653 | 44 / 36 | 0	1691 | 54 / 36 | 0	1717 | 62 / 36 | 0	1751 | 70 / 36 | 0	1853 | 78 / 36 | 0	2017 | 88 / 36 | 0	3463 | 96 / 36 | 0
6	2025 | 44 / 52 | 22	2063 | 54 / 52 | 22	2089 | 62 / 52 | 22	2123 | 70 / 52 | 22	2225 | 78 / 52 | 22	2389 | 88 / 52 | 22	3835 | 96 / 52 | 22
4	2286 | 44 / 68 | 48	2293 | 46 / 68 | 48	2350 | 62 / 68 | 48	2384 | 70 / 68 | 48	2486 | 78 / 68 | 48	2650 | 88 / 68 | 48	4096 | 96 / 68 | 48

Change in DO Standards, mg/ℓ

KEY: System Cost | Clayton Level / McDonough & Atkinson Level | Yates Level

Figure 45. Optimal System Cost and Abatement Policy as a Function of TRS and DOS for TMS = 91°F

Temperature Rise Standard, °F	-3		-2		-1		0		+1		+2		+3	
16	1140	44	1178	54	1204	62	1238	70	1340	78	1504	88	3172	96
	0	0	0	0	0	0	0	0	0	0	0	0	12	0
14	1140	44	1178	54	1204	62	1238	70	1340	78	1504	88	3172	96
	0	0	0	0	0	0	0	0	0	0	0	0	12	0
12	1187	44	1225	54	1251	62	1285	70	1387	78	1551	88	3172	96
	2	0	2	0	2	0	2	0	2	0	2	0	12	0
10	1478	44	1516	54	1542	62	1576	70	1678	78	1842	88	3288	96
	20	0	20	0	20	0	20	0	20	0	20	0	20	0
8	1653	44	1691	54	1717	62	1751	70	1853	78	2017	88	3463	96
	36	0	36	0	36	0	36	0	36	0	36	0	36	0
6	2025	44	2063	54	2089	62	2123	70	2225	78	2389	88	3835	96
	52	22	52	22	52	22	52	22	52	22	52	22	52	22
4	2286	44	2293	46	2350	62	2384	70	2486	78	2650	88	4096	96
	68	48	68	48	68	48	68	48	68	48	68	48	68	48

Change in DO Standards, mg/ℓ

KEY:

System Cost	Clayton Level
McDonough & Atkinson Level	Yates Level

Figure 46. Optimal System Cost and Abatement Policy as a Function of TRS and DOS for TMS = 93°F

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